An Algorithm for Constructing Virtual Backbone Network Under Routing Cost Constraint for Heterogeneous Wireless Sensor Networks

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Yuetao Wang, Chunyan Liu, Yunlong Zhao, Yang Li
College of Computer Science and Technology
Nanjing University of Aeronautics and Astronautics
Nanjing, China
{wangyuetao, lcy_cs, zhaoyunlong, liyangnuaa}@nuaa.edu.cn

***Abstract*—In order to avoid broadcast storms in wireless sensor networks and improve the efficiency of information transmission, virtual backbone networks have been widely introduced into network information transmission. In order to combine the practical development needs, this paper devotes to the optimization research on the construction of virtual backbone network, optimizes the three aspects of routing path length, backbone size and fault tolerance respectively, introduces the GOC-CDS construction strategy and the αMOC-SCDAS construction strategy, and designs an approximation algorithm for the construction of the virtual backbone network based on the two strategies. Through theoretical analysis, the algorithm can obtain a $(72\rho^2 + 24\rho + m + 1)(2\rho + 1)^2$-approximate optimal solution, where $\rho = \frac{r_{max}}{r_{min}}$. After comparative experiments, it is verified that the algorithm can maintain a smaller routing path length of the network and a better fault tolerance of the virtual backbone network, and at the same time, it can ensure that the size of the backbone network is as small as possible.**

***Index Terms*—Virtual backbone network, Connected dominating set, Fault tolerance, Routing cost constraint, Heterogeneous wireless sensor network.**

I. INTRODUCTION

Wireless sensor networks (WSNs) , with many features that are different from traditional wired networks, have been widely used in areas where it is difficult to install base stations in the network. Since there is no fixed infrastructure, in order to effectively address the high variability of network topology, reduce unnecessary energy consumption caused by redundant routing, and avoid information conflicts and broadcast storms, Ephremides et al. [1] first introduced the concept of Virtual Backbone Network (VBN). The message interactions between any nodes in the network can be transformed into homologous operations on VBN, which greatly reduces the message forwarding process, shortens the routing path search time to a certain extent, reduces the routing table, and simplifies the routing maintenance. In WSNs, the VBN construction method based on the Connected Dominating Set (CDS) is a common and competitive approach.

In practical scenarios, sensors in WSNs can have varying transmission ranges due to factors such as power and transmission medium. This results in a network known as a heterogeneous WSN, it can be modeled as a directed graph where nodes have different transmission radius. Constructing a virtual backbone network in a heterogeneous WSN can then be transformed into the problem of constructing a Strongly Connected Dominating and Absorbing Set (SCDAS) in graph theory. Constructing an efficient virtual backbone has become a research hotspot in the academic community.

Kim et al. [2] proposed another concept named Average Backbone Path Length (ABPL) to evaluate the routing length. However, researches have found that the ABPL of networks using virtual backbones is often much greater than the original path length, significantly increasing communication delay and routing overhead. To ensure that the routing path between any two points in VBN-based communication is as short as possible, researchers have proposed virtual backbones with routing cost constraint. By adding some backbone nodes, this approach ensures that the routing path length between any pair of nodes in the network does not exceed a constant multiple α of the shortest path length in the original network. This method reduces energy consumption and transmission delay. Researches have found that this approach can achieve a balance between routing cost constraint and the size of the backbone network to some extent.

Since sensor networks are often deployed in harsh or hostile environments, channel bandwidth frequently varies and connectivity can be intermittent. Sensors often fail due to energy depletion or damage. Consequently, topologies designed for optimal energy efficiency in theory are often unsuitable for practical applications. In practical applications, a reasonable topology should retain some redundancy to cope with potential channel and node failures, thereby ensuring a certain degree of fault tolerance. Thus, the study of fault-tolerant virtual backbones has garnered considerable attention to enhance the robustness, resilience, and availability of the network. In graph theory, this problem can be addressed by constructing a (k, m)-CDS, which requires that each node outside the virtual backbone is dominated by at least m nodes within the virtual backbone, and there are at least k disjoint paths between any pair of backbone nodes. This means that even if $k - 1$ nodes fail, the virtual backbone can still function normally.

Based on the need to optimize routing path length in heterogeneous networks as well as to improve the robustness of virtual backbones, this paper proposes the problem of constructing virtual backbones in heterogeneous WSNs while simultaneously considering routing path length and fault tolerance of the backbone. This problem is transformed into the (k,m)-α Minimum rOuting Cost Strongly Connected Dominating and Absorbing Set ((k,m)-αMOC-SCDAS) problem and an approximate construction algorithm is provided, which uses routing cost constraint to reduce the communication path length between nodes and enhances the fault tolerance of the virtual backbone by constructing a (k,m)-SCDAS. Through theoretical analysis and simulation experiments, it is verified that the target set generated by the algorithm effectively reduce redundant paths, save energy consumption, and also improve the fault tolerance of the backbone to a certain extent, prolonging the network lifespan.

II. RELATED WORK

A. Strongly Connected Dominating Set

For a directed disk graph with unidirectional edges, its virtual backbone can be modeled as a SCDS. Dai et al. [3] proposed a local algorithm to construct a SCDS using a marking process. Du et al. [4] proposed two algorithms to construct the SCDS, using a Breadth First Search (BFS) tree and the Minimum Steiner Number (MSN), respectively. Chou et al. [5] proposed a centralized algorithm to construct a SCDS, extending the results of [4]. The virtual backbone constructed by their algorithm can provide efficient bidirectional data transmission services between any two sensor nodes in the network. Park et al. [6] proposed a constant approximation algorithm and two heuristic approximation algorithms, obtaining an upper bound of $9.6(\rho+12)^2opt+14.8(\rho+12)^2$ for the Minimum Strongly Connected Dominating and Absorbing Set (MSCDAS) problem, this was the first study on the MSCDAS problem. Wu [7] extended the concept of the dominating set (DS) in unit disk graphs (UDG) to the dominating and absorbing set (DAS) in disk graphs (DG), and proposed a local algorithm for constructing a SCDAS. Li et al. [8] presented a $(3H(n-1)-1)$-approximation algorithm for the MSCDAS problem without making any assumptions about transmission radius. This algorithm is applicable to the MSCDAS problem in any directed graph. Later, Zhang et al. [9] proposed a $(2+\epsilon)$-approximation algorithm for the minimum DAS problem, which was the first algorithm to achieve a constant approximation ratio for DAS, they also proposed a $(4+3ln(2+\epsilon)opt+\epsilon)$-approximation algorithm for the MSCDAS problem.

B. Virtual Backbone with Guaranteed Routing Cost

In 2007, Li et al. [10] began studying the problem of constructing diameter-bounded VBN in WSNs and proposed a heuristic algorithm for constructing diameter-bounded CDS with an approximation ratio of $11.4|\text{MCDS}|+1.6$. Kim et al. [2] proposed an approximation algorithm for constructing VBN using the minimum spanning tree algorithm, called CDS-BD-C2. This algorithm can generate a CDS with bounded backbone size and average backbone path length in undirected graphs with the same node transmission radius, with an approximation ratio of 6.906.

In 2010, Ding et al. [11] introduced the concept of minimum routing cost virtual backbone (MOC-CDS). Routing via MOC-CDS ensures that every routing path between any pair of nodes is also the shortest path in the original network, thus significantly reducing energy consumption and latency, the approximation ratio of this algorithm is $(1-ln2)+2ln\delta$, δ is the maximum node degree in the network. Through simulation experiments, it was found that the number of backbone nodes in MOC-CDS was too large. In order to strike a balance between VBN size and routing path length, Ding et al. [12] proposed the α minimum routing cost connected dominating set (α-MOC-CDS) problem in 2011. In 2016, Liu et al. [14] studied the problem of α-MOC-SCBDS (α minimum routing cost constraint strongly connected bidirectional dominating set) for heterogeneous networks with different transmission radius for each node and proposed an approximation algorithm. Later, Putwattana et al. [15] improved the algorithm of Liu [14] by further reducing the constraint value of connectivity paths. In 2016, Zhang et al. [16] studied the calculation of MSCDAS problem under routing cost constraint, proposing a polynomial-time approximation algorithm. In 2020, Liang et al. [17] proposed two algorithms to solve the diameter-bounded MSCDAS problem, called GOC-SCDAS and α-GOC-SCBDAS, the experimental results outperform the algorithm of Du et al. [4]. For general graphs, the known approximability result is the research by Ding et al. [18], who proposed several approximation algorithms for different values of α, improving upon the approximation by Du et al. [19].

C. Virtual Backbone with Fault Tolerance

Dai and Wu [20] were the first to introduce the fault-tolerant virtual backbone problem into the study of VBN. In 2007, Shang et al. [21] introduced the concept of k-connected m-dominating sets ((k,m)-CDS), where $m>k$, and proposed three approximation algorithms.

In [22], Wang et al. presented an approximation algorithm for constructing a (2, 1)-CDS with an approximation ratio of 62.19. In [23], Shi et al. proposed a greedy-based algorithm for constructing (2, m)-CDS and also pointed out an error in the approximation ratio analysis of the (2, m)-CDS construction algorithm by Shang et al. [21]. In 2013, Wang et al. [24] first provided an approximation construction algorithm FT-CDS-CA for (3, m)-CDS. They demonstrated that the approximation ratio of this algorithm is 280 for constructing (3, 3)-CDS, $21r$ for constructing (3, m)-CDS ($k>3$), and $r=5+\frac{25}{m}$ when $3\le k\le 5$, and $r=11$ when $k>5$. In [25], Liu et al. proposed an algorithm for (3, m)-CDS construction with an even smaller approximation ratio using Tutte decomposition in graph theory. In 2017, Wang et al. [26] introduced an approximation algorithm that can be used for constructing (3, m)-CDS and (4, m)-CDS, and they provided the approximation ratio of the algorithms. The above approximation algorithms are all based on a specific value of k. For constructing (k,m)-CDS for any value of k ($2 \leq k \leq m$), researchers such as Shi et al. [27], Fukunaga [28], and Liu et al. [29] have conducted research and proposed approximation algorithms.

The problem of constructing fault-tolerant VBN in heterogeneous WSNs was first introduced by Tiwari et al. [30], where they transformed the problem into finding the (k, m)-SCDAS problem in DG and gave the appropriate solutions. In [31], Thai et al. transformed the problem into solving the (k, m)-CDS problem in bidirectional link disk graphs, and they proposed a $(\theta + ln\theta + m + 2)(2k - 1)$-approximation algorithm, where θ represents the maximum number of independent neighboring nodes for any node. In [32], Tiwari et al. introduced a construction algorithm for (1, m)-SCDAS with an approximation ratio of $2(\theta(1 + \frac{1}{m}) + m + 1)$. Therefore, as of now, there are no known approximation algorithms for constructing (k, m)-SCDAS.

III. PROBLEM STATEMENT

A. Network Model

In wireless sensor networks, the transmission range of all sensor nodes is uncertain. For any pair of nodes u, v, if node v is within the transmission range of node u, it is assumed that node v can accept messages from node u. Typically, the above network can be described using a directed graph, where V represents sensor nodes and E denotes whether direct communication between sensor nodes is possible. For any pair of nodes u and v, if $u, v \in V$, it indicates that node v can receive messages from node u; if $(u, v) \in E$ and $(u, v) \in E$, it means u and v can exchange messages with each other.

B. Problem Definition

Given a directed graph $G(V, E)$ and a set of nodes $S \subseteq V$, for any node $u \in V$, either $u \in S$, or there exists nodes $v, w \in S$ such that $(u, v) \in E$ and $(v, w) \in E$, then the set S is called a dominating and absorbing set of the graph G. Let $G[S]$ denote the induced subgraph by S, if for any pair of nodes $(u, v) \in S$, there exists a directed path from u to v in $G[S]$, then $G[S]$ is said to be strongly connected.

For a CDS, for any pair of nodes $u, v \in V$, let $p(u, v)$ represent the shortest path from u to v in G, and let $d(u, v)$ represent the length of $p(u, v)$. Similarly, let $p_S(u, v)$ represent the shortest path from u to v in $G[S]$, and let $d_S(u, v)$ represent the length of $p_S(u, v)$.

This paper considers the construction problem of (1, m)-αMOC-SCDAS in directed DGs. Clearly, (1, m)-αMOC-SCDAS is a type of SCDAS under specific constraint. Therefore, the (1, m)-αMOC-SCDAS problem can be formally defined as follows:

Definition 1. ***(1, m)-αMOC-SCDAS:*** *Given a directed DG G, a subset S is called a fault-tolerant α-routing cost constrained SCDAS based on a m-DAS if it satisfies the following conditions:*
1) *S is a SCDAS of G.*
2) *For a given constant $\alpha \geq 1$, $\forall u, v \in V$, $d_S(u, v) \leq \alpha \times d(u, v)$.*
3) *All regular nodes in G are dominated and absorbed by at least m backbone nodes in S.*

The target set construction problem proposed by this definition is aimed at directed graphs with unidirectional edges., where the transmission radius of nodes are unequal. This construction method differs from those used to construct CDS in undirected graphs, the transmission radius of nodes is a key factor in selecting nodes. Message routing through the backbone composed of the target node set can ensure message interoperability between any pair of nodes in the network, with routing path length constrained within a certain range, and the backbone itself possesses a degree of fault tolerance.

IV. ALGORITHM DESIGN AND THEORETICAL ANALYSIS

A. Strategy and Algorithm Design

Firstly, in order to address the issue of excessively long routing paths in networks routed through VB, we adopt the approach of routing cost constraint to reduce redundant paths. The problem of constructing a VBN with routing cost constraint in graph theory is addressed by selecting a α-MOC-CDS. Existing research has demonstrated that finding a α-MOC-CDS is NP-hard. Some literatures address this problem by proposing approximate algorithms for finding α-MOC-CDS. Below, we provide theoretical support for this approach through some strategies.

According to Lemmas 1$\sim$4 in [13] and the theoretical proof of the improvement of routing constraint values in [15], we introduce the construction strategy for Guaranteed Routing Cost Connected Dominating Set (GOC-CDS) as follows:

GOC-CDS Construction Strategy: In a connected graph G, after obtaining a DS C of G , include all nodes from the set C into S. For any pair of nodes $u, v \in S$, if $d(u, v) \leq 3$, add all nodes on the shortest path between them to the set S, then for $\forall u, v \in V$, $d_S(u, v) \leq 5d(u, v)$.

To enhance the fault tolerance of the VBN, we introduce m-DAS to construct αMOC-SCDAS, with the specific strategy as follows:

αMOC-SCDAS Construction Strategy: Building upon the GOC-CDS construction strategy, we solve the minimum DAS $I_1, I_2, ..., I_m$ of the set of regular nodes through an iterative method and add them to the GOC-CDS. Through $m-1$ iterations, we obtain the m-dominating αMOC-SCDAS, ensuring that every regular node is dominated and absorbed by at least m backbone nodes.

Based on the GOC-CDS construction strategy and the αMOC-SCDAS construction strategy, we devise a three-stage (1, m)-αMOC-SCDAS construction algorithm:

1) Stage One: Construct a minimum DAS as the initial backbone node set.
2) Stage Two: Generate a SCDAS based on the GOC-CDS construction strategy.
3) Stage Three: Generate the m-dominating αMOC-SCDAS based on the αMOC-SCDAS construction strategy.

B. Detailed Algorithm Design

Inspired by Algorithm 1 in [18], we designs an algorithm for constructing the minimum DAS of a graph. The algorithm draws on the idea of the tricolor mark-and-sweep algorithm, where initially, all nodes are colored white, during the execution of the algorithm, the selected backbone nodes are colored as black, and their neighbours are colored as gray. The specific algorithm process is outlined in Algorithm 1. The algorithm employs a greedy strategy to greedily select nodes with the maximum transmission range in the node set as backbone nodes, ultimately forming a minimum DAS. For a directed graph G, the direction of all directed edges is reversed to obtain a directed graph G' with the opposite direction of edge sets, as indicated in the first line of Algorithm 1. In the first stage, the algorithm proceeds by applying the greedy strategy twice, separately operating on the graph G and G', to generate a minimum DAS C for the original directed graph G.

Algorithm 1 Construct DAS(G, Radius) algorithm

Input: A connected directed graph $G = (V, E)$ with unidirectional links.
Output: A DAS C of G

1: Generate a directed graph G' by reversing the edges of graph G, and color all nodes in both graphs G and G' white
2: $C \leftarrow \emptyset$
3: **while** There is a White node in G **do**
4: Select a White node u having maximum Radius of White nodes in G and color it Black
5: Color all the White nodes in $N^+(u)$ Gray
6: $C \leftarrow C \cup \{u\}$
7: **end while**
8: **while** There is a White node in G' **do**
9: Select a White node v having maximum Radius of White nodes in G' and color it Black
10: Color all the White nodes in $N^+(v)$ Gray
11: $C \leftarrow C \cup \{v\}$
12: **end while**
13: **return** C

In the second stage of the (1, m)-αMOC-SCDAS algorithm, it is necessary to construct a connected node set to connect the DAS obtained in the first stage. In this stage, Strategy on routing cost constraint is implemented. The description of the second stage of the algorithm is illustrated in Algorithm 2.

Algorithm 2 Construct Connected Set(G, C) algorithm

Input: A connected directed graph $G = (V, E)$ with unidirectional links, and a DAS C of G.
Output: A connected set D of G.

$D \leftarrow \emptyset$
$D \leftarrow C$
while There exist a pair of nodes $u, v \in C$ which satisfy $d(u, v) \leq 3$ **do**
Find a shortest path p between the nodes $u, v \in C$
$D \leftarrow D \cup p \setminus \{u, v\}$
end while
return D

Algorithm 2 connects the DAS to form a SCDAS according to the GOC-CDS construction strategy. It creates an initial node set D, then computes a shortest path in G for any pair of nodes $u, v \in C$ satisfying $d(u, v) \leq 3$, and includes all intermediate nodes along the path in the set D, the final obtained set D is a connected node set. Then, the union of the minimal DAS C constructed in the first stage and the connected node set D forms a SCDAS for the graph G.

The third stage extends the current SCDAS to m-dominating and absorbing through an iterative method. It uses a FOR loop for $m - 1$ iterations, where each iteration calls Algorithm 1 to generate the minimal DAS I_i of the current graph, where $i = 2, 3, ..., m$. Finally, taking $S = S \bigcup I_2 \bigcup I_3 \bigcup ... \bigcup I_m$ forms a (1, m)-αMOC-SCDAS for the graph G. The detailed description of the third stage of the algorithm is illustrated in Algorithm 3.

Algorithm 3 $(1, m)$-αMOC-SCDAS algorithm

Input: A connected directed graph $G = (V, E)$ with unidirectional links.
Output: A $(1, m)$-αMOC-SCDAS S of G.

$C \leftarrow$ Construct DAS(G, Radius)
$D \leftarrow$ Construct Connected Set(G, C)
$S \leftarrow C \cup D$
The set S is the SCDAS of G
for $i = 1$ **to** $m - 1$ **do**
Color all the Gray nodes in G and G' White
$S \leftarrow S \cup$ Construct DAS(G, Radius)
end for
The set S is the $(1, m)$-αMOC-SCDAS of G
return S

C. Theoretical Analysis and Approximation Ratio Calculation of Algorithm

Algorithm 3 extends the target node set to a (1, m)-αMOC-SCDAS through an iterative approach. The final selected node set still satisfies the corresponding routing constraint. The specific theoretical analysis process is provided in Theorem 1 as follows.

Theorem 1. *Let C be the minimal DAS constructed in the first stage of the algorithm, and let S be αMOC-SCDAS constructed in the third stage of the algorithm. For any node $u \in V$, either it belongs to the set S or it is dominated and absorbed by at least m nodes in the set S; and $\forall u, v \in V$, we have*

$$d_S(u, v) \leq 5d(u, v) \tag{1}$$

Proof. First, we prove that the set constructed in the third stage of the algorithm is strongly connected. After the third stage of the algorithm, since the $(m-1)$-SCDAS is strongly connected before the $m-1$ iterations, the remaining nodes are all in a state of being dominated and absorbed, therefore, adding any new backbone node can still ensure strong connectivity. Going back to before the first iteration, the first and second stages of the algorithm have already constructed a (1, 1)-SCDAS, thus, the remaining nodes must have dominating and absorbing neighbor nodes. After the first iteration, adding some backbone nodes ensures that the set now satisfies 2-dominating and absorbing. These newly added nodes were previously dominated and absorbed nodes, so the current (1, 2)-SCDAS still ensures strong connectivity.

Next, we prove that the set constructed after the third stage of the algorithm satisfies the property of m-dominating and absorbing. This is evident because after each iteration of the DAS construction algorithm, every regular node in the network will be dominated and absorbed by at least one additional backbone node. After $m-1$ iterations, the remaining non-backbone nodes in the network will be dominated and absorbed by at least m backbone nodes.

Furthermore, we prove that the set constructed by the algorithm is still a αMOC-SCDAS. It is known that the (1, 1)-MOC-SCDAS constructed in the second stage of the algorithm in this paper satisfies $d_S(u,v) \leq 5d(u,v)$. Since regardless of how many iterations of the DAS construction algorithm are performed, the newly added backbone nodes are already dominated and absorbed, the SCDAS constructed in the first and second stages of the algorithm is a αMOC-SCDAS. Therefore, any pair of nodes can ensure the routing cost constraint of α, and the iterative method of constructing multiple-DAS only adds some backbone nodes to increase the fault-tolerance of VB without affecting the routing cost constraint.

Next, we use the concept of circle packing [33] and introduce virtual disks into the directed graph , whose radius is $\frac{r_{min}}{2}$, with the sensor nodes as the disk centers and the radius $\frac{r_{min}}{2}$ as the transmission ranges. Since there are no real nodes with a transmission radius of $\frac{r_{min}}{2}$ in $G(V,E)$, it is virtual. However, this virtual disk plays an important role in the proof of the theorem for calculating the approximation ratio of the algorithm.

Lemma 7 in [34] obtaining an upper bound on the number of the independent neighbors of the node, then we can calculate the upper bound of the minimal DAS generated in the first stage of the algorithm. Putwattana et al. [15] conducted theoretical analysis and computed the upper bound of the size of SCBDS constructed by their algorithm. Following their approach, we also conducted theoretical analysis and computed the upper bound of the size of SCDAS constructed by our algorithm, as detailed in Lemma 1.

Lemma 1. *If a SCDAS is constructed by connecting the nodes in C, where all intermediate nodes on a shortest path between any pair of nodes in C are added to the set S, i.e., connected by a path of length at most 3 hops, then the size of S is at most*

$$2(6\rho+1)^2(2\rho+1)^2 opt. \tag{2}$$

Finally, we can calculate the upper bound of the target node set constructed by our (1, m)-αMOC-SCDAS algorithm, as stated in Theorem 2.

Theorem 2. *The upper bound of the (1, m)-αMOC-SCDAS is*

$$(72\rho^2+24\rho+m+1)(2\rho+1)^2 opt \tag{3}$$

where $\rho = \frac{r_{max}}{r_{min}}$, and opt is the optimal solution for the MSCDAS problem.

Proof. According to the basic idea of the third stage of the algorithm, the final (1, m)-αMOC-SCDAS is obtained by combining the (1, 1)-αMOC-SCDAS generated in the second stage and $(m-1)$-DAS obtained by iterative method. From Lemma 1, the upper bound of the (1, 1)-αMOC-SCDAS is approximately $2(6\rho+1)^2(2\rho+1)^2 opt$. Then, combining Lemma 7 in [34] and the above Lemma 1, we can calculate that the upper bound of the (1, m)-αMOC-SCDAS is $2(6\rho+1)^2(2\rho+1)^2 opt + (m-1)(2\rho+1)^2 opt = (72\rho^2+24\rho+m+1)(2\rho+1)^2 opt$.

V. EXPERIMENTS AND ANALYSIS

A. Simulation Environment and Comparison Algorithms

We reproduced the relevant algorithms on MATLAB and conducted experiments on a personal computer equipped with a 3.50 GHz 13th Gen Intel(R) Core(TM) i5-13600KF processor and 32 GB of RAM.

In the simulation, we model a wireless sensor network as a set of nodes randomly deploy in a 100×100 Euclidean plane. The number of nodes varies among 100, 150, 200, 250, 300, 350 and 400. Each node has a fixed transmission range in the range of $[r_{min}, r_{max}]$. r_{min} is set to 15 and 25, and $r_{max} = \rho r_{min}$ where ρ varies among 1.25, 1.50, and 1.75. In each simulated network, a directed edge from node u to node v exists if and only if v is within the transmission range of u. The directed DG corresponding to the simulated network is then constructed according to the method described in Section 2, and it is checked whether the graph is strongly connected. If the graph is strongly connected, it is retained as a candidate network graph; otherwise, the graph is discarded, and the network and graph are regenerated. In order to avoid chance, for each set of parameter settings (e.g., number of nodes equals 100, r_{min} = 15, ρ = 1.25), 20 simulated networks are generated randomly. Each kind of algorithm is tested 10 times on each simulated network, and the average result is taken as the performance of the algorithm under that set of parameter settings. For the comparison of the fault tolerance of the VB constructed by different algorithms, each simulated network is randomly disrupted by 10%, 20%, and 30% of its nodes. Finally, the size of the VB, the average backbone path length (ABPL), and the success rate of maintaining backbone performance after partial node failures are used for comparing the simulation results.

There has been extensive research on the MCDS problem. However, there are few studies on the MSCDAS problem that incorporates routing cost constraint and fault tolerance. Upon examination, CDS-BD-C2 [2] can generate a SCDS in directed graphs with unidirectional edges, then we improve this algorithm to SCDAS-BD-C2 to satisfy the SCDAS problem, and included it as one of the comparison algorithms in our study. CDS-BFS [4] generates a SCDS that only ensures bidirectional communication between backbone nodes. We have modified it to generate a SCDAS that ensures bidirectional communication between backbone nodes and regular nodes as well. The modified algorithm is named SCDAS-BFS and is compared with the (1, m)-αMOC-SCDAS algorithm in our study.

Additionally, to address the problem of overly long paths when routing communication through a VB, applying routing cost constraint is a common solution. A good method for this optimization goal is to select the intermediate nodes on the shortest paths between all node pairs in the network as backbone nodes. This approach was proposed by Ding et al. [18], but they did not provide a good centralized algorithm. Therefore, we designed a SCDAS construction algorithm based on the shortest paths in the network, named ShortestPath-SCDAS, as one of the comparison algorithms for the (1, m)-αMOC-SCDAS algorithm.

B. Comparison and Analysis of Simulation Results

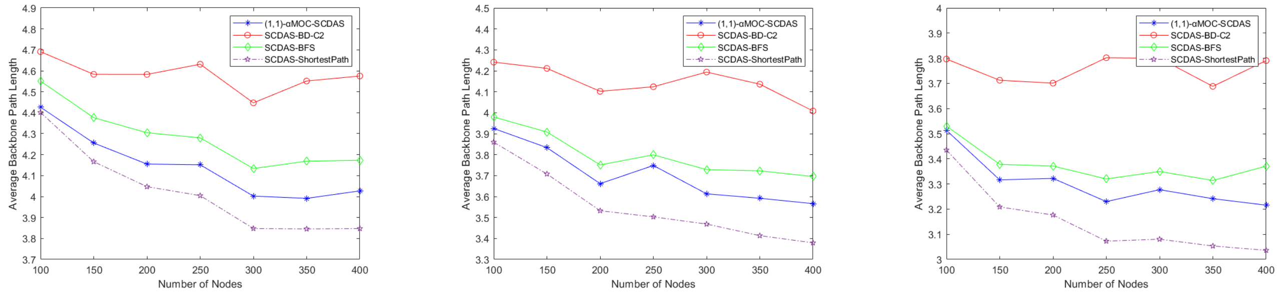
In this section, we conduct experiments to compare and analyze the effectiveness of the algorithm (1, m)-αMOC-SCDAS with SCDAS-BD-C2, SCDAS-BFS , and ShortestPath-SCDAS. We evaluate the SCDAS generated by each algorithm using three metrics: (1) ABPL, (2) SCDAS size, and (3) SCDAS fault tolerance.

Figure 1 shows the performance of (1, m)-αMOC-SCDAS, SCDAS-BD-C2, SCDAS-BFS, and ShortestPath-SCDAS in terms of ABPL with r_{min} picked 15 and ρ picked 1.25, 1.50, and 1.75, respectively. Homogeneously, Figure 2 presents the experimental results with $r_{min} = 25$. As the figures show, ABPL slightly decreases with the increase of the number of nodes. It is reasonable because the shortest path of some pair of nodes may decrease with the increase of the number of nodes. While the number of nodes is small, the shortest path of a pair of nodes may need many intermediate nodes. However, while the number of nodes is large enough, the Euclidean distance of the shortest path of a pair of nodes is close to the Euclidean distance of the two nodes. Therefore, the length of the shortest path will not decrease evidently while the number of nodes is large enough. Additionally, Figures 1 and 2 demonstrate that ABPL of the VB generated by (1, m)-αMOC-SCDAS is slightly larger than that of ShortestPath-SCDAS, which uses the shortest path in original graph. However, it is smaller than that of the two algorithms that do not consider routing cost constraint.

Figure 3 shows the performance of the four algorithms in terms of SCDAS size with r_{min} picked 15 and ρ picked 1.25, 1.50, and 1.75, respectively. Homogeneously, Figure 4 presents the experimental results with $r_{min} = 25$. From Figures 3 and 4, the average size of SCDAS slightly increases with the number of nodes for all algorithms. This situation arises because as the total number of nodes increases, any algorithm will need to add more backbone nodes to construct the SCDAS during its execution. Our algorithm has a larger SCDAS size than SCDAS-BFS and SCDAS-BD-C2 . However, its has a smaller ABPL than these two algorithms, and significantly smaller compared to ShortestPath-SCDAS, which directly utilizes the shortest path to connect backbone nodes. Anyway, the SCDAS size is bounded in our algorithm as Theorem 2 shows.

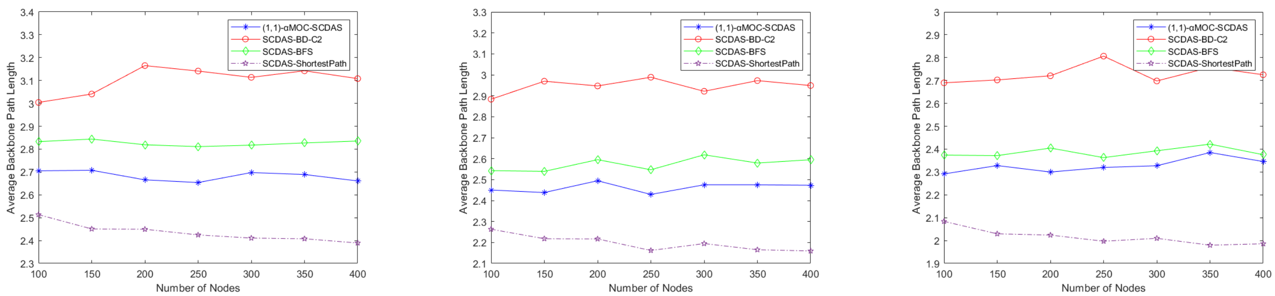
The third evaluation criterion is the fault tolerance performance of the VB generated by different algorithms. Setting the random range of node transmission radius to [15, 30], and then randomly disrupting 10%, 20%, and 30% of the nodes for each simulated network, the probabilities of the VB constructed by different algorithms to maintain connectivity are calculated. Figure 5 shows the fault tolerance performance of the VB constructed by (1, 3)-αMOC-SCDAS, (1, 2)-αMOC-SCDAS, (1, 1)-αMOC-SCDAS, SCDAS-BD-C2, and SCDAS-BFS when the percentage of randomly disrupted nodes is 10, 20, and 30, respectively, it intuitively reflects the success rate of the remaining backbone to maintain backbone performance after random node disruptions, two criteria are set for detecting backbone performance: (1) whether the remaining VBN is strongly connected, and (2) whether regular nodes are dominated by at least one backbone node and absorbed by one backbone node at the same time. From Figure 5, it can be observed that in the random input graphs with the total number of sensor nodes ranging from 100 to 400, as the number of disrupted nodes increases from 10% to 30%, the success rates of maintaining backbone performance of the SCDAS constructed by various algorithms all decrease. This changing trend is evident because the fault tolerance of the VB constructed by the algorithm is limited. As the number of randomly disrupted nodes increases, the number of original backbone nodes decreases accordingly, ultimately some regular nodes may not be dominated by backbone nodes. For construction algorithms that do not consider backbone fault tolerance, they can only guarantee that regular nodes are dominated or absorbed by at least one backbone node. When a backbone node fails due to various factors, regular nodes with only one backbone neighbor lose communication with the network. This leads to the inability of the current backbone to maintain VB characteristics, and the remaining VBN cannot function properly. Therefore, it is necessary to consider fault-tolerant VB construction strategies, and our (1, m)-αMOC-SCDAS algorithm optimizes the fault tolerance of VB construction.

For (1, m)-αMOC-SCDAS, Figure 5 shows the experimental results with m picked 1, 2, or 3, compared with the effects of SCDAS-BD-C2 and SCDAS-BFS. It can be observed that (1, m)-αMOC-SCDAS demonstrates a significantly higher success rate in maintaining backbone performance after randomly disrupting 10%, 20%, and 30% of the network nodes



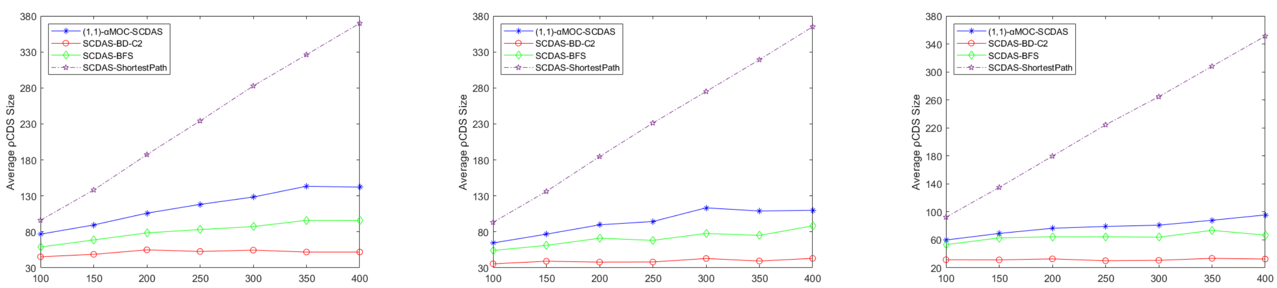
(a) $\rho = 1.25$ (b) $\rho = 1.50$ (c) $\rho = 1.75$

Fig. 1. ABPL versus node number with $r_{min} = 15$



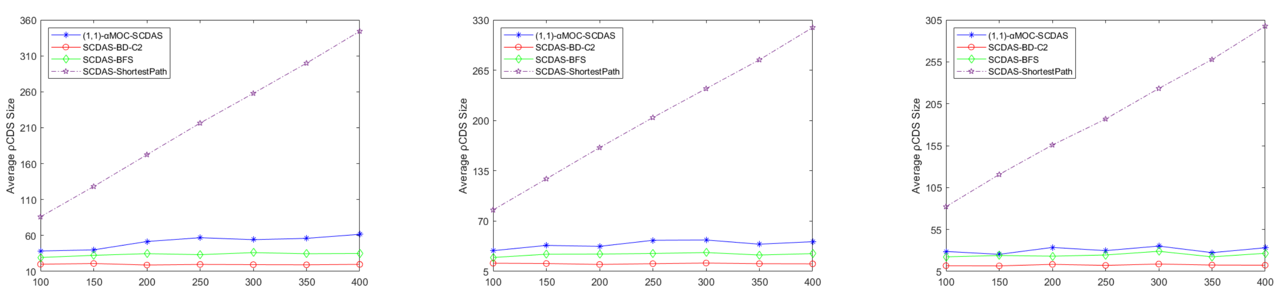
(a) $\rho = 1.25$ (b) $\rho = 1.50$ (c) $\rho = 1.75$

Fig. 2. ABPL versus node number with $r_{min} = 25$



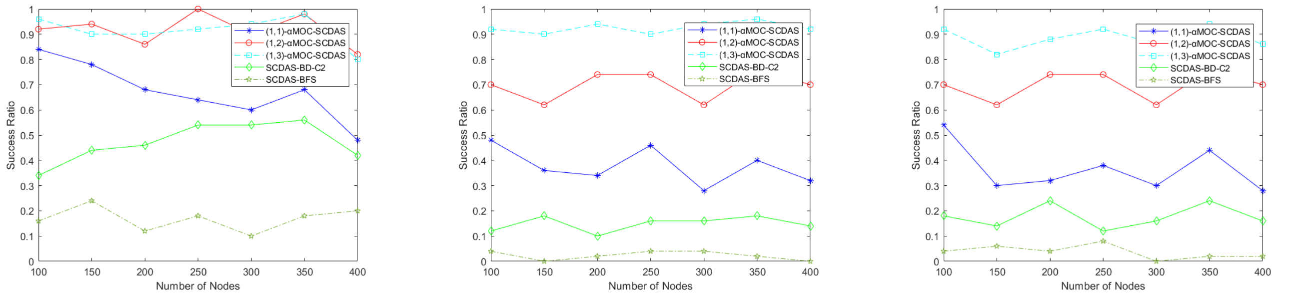
(a) $\rho = 1.25$ (b) $\rho = 1.50$ (c) $\rho = 1.75$

Fig. 3. SCDAS size versus node number with $r_{min} = 15$



(a) $\rho = 1.25$ (b) $\rho = 1.50$ (c) $\rho = 1.75$

Fig. 4. SCDAS size versus node number with $r_{min} = 25$



(a) The number of failed nodes accounts for 10% (b) The number of failed nodes accounts for 20% (c) The number of failed nodes accounts for 30%

Fig. 5. Success Rate of Remaining VBN Maintaining Backbone Performance After Disrupting a Portion of Nodes

compared to the other two algorithms. Additionally, as m increases, the success rate of maintaining backbone performance improves notably. This is because any regular node is dominated and absorbed by more than one backbone node, if one neighboring backbone node fails, the remaining m-1 neighboring backbone nodes can still maintain the backbone performance, ensuring the entire VBN continues to function properly.

In summary, in a heterogeneous WSN with varying transmission radius for sensor nodes, our algorithm (1, m)-αMOC-SCDAS effectively addresses the construction of a VBN with fault tolerance and routing cost constraint. This algorithm enhances the robustness of the network and optimizes the routing path length between nodes. As a result, the path length for communication through the VB is significantly reduced, while the fault tolerance of the VB is improved to a certain extent. Compared to the other algorithms, (1, m)-αMOC-SCDAS produces a SCDAS with a smaller backbone size, shorter average backbone path length, and higher fault tolerance. Overall, it demonstrates the best comprehensive performance.

VI. CONCLUISON

The construction and optimization of virtual backbone networks have been a research focus for a long time. To address the construction of virtual backbone networks in heterogeneous environments with varying node transmission ranges, we designed a virtual backbone network construction scheme that simultaneously optimizes the routing path length and the fault tolerance of the backbone network, and proposed the (1, m)-αMOC-SCDAS algorithm based on a strongly connected dominating and absorbing set. Simulation experiments show that the target node set generated by our algorithm results in significantly lower routing overhead compared to SCDAS-BD-C2 and SCDAS-BFS, and it offers substantially higher fault tolerance. And as mentioned earlier, for any pair of nodes u and v, we have $d_S(u,v) \leq 5d(u,v)$. Additionally, our algorithm can achieve a $(72\rho^2 + 24\rho + m + 1)(2\rho + 1)^2$-approximate optimal solution, where $\rho = \frac{r_{max}}{r_{min}}$.

The experiments in the paper were conducted in a simulated environment. However, real-world networks are influenced by various factors such as dynamic changes in nodes, signal interference, and privacy protection. In our study, routing information is utilized to establish a virtual backbone network. However, in wireless networks, routing cost is usually unstable. Then, how to construct a virtual backbone network based on these problems that will be encountered in the real situation is a problem that needs to be considered in our subsequent research.

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