Modeling Friction in Smooth Multibody Systems in Absolute Coordinates

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Abstract

In this work we propose a method to simulate friction in continuous multibody systems with $\mathbf{r}-\mathbf{A}$ formulation. The $\mathbf{r}-\mathbf{A}$ formulation, works directly with the orientation matrix and therefore avoids the need for Euler parameters used to produce the orientation matrix $\mathbf{A}$, by using Lie-Algebraic integration. The frictionless systems have been recently studied in detail with $\mathbf{r}-\mathbf{A}$ formulation in [1]. Further, Haug in [2], describes the equations to model friction in different mechanical joints in $\mathbf{r}-\mathbf{p}$ formulation, by resolving contact forces assuming static equilibrium. In the $\mathbf{r}-\mathbf{A}$ formulation, the torques produced by external forces act on the center of mass. Hence, the resolution of contact forces can be done under dynamic equilibrium. This work explores the same and investigates implementation of friction in $\mathbf{r}-\mathbf{A}$ formulation. To the best of our knowledge, the results of such a study have not been reported in the literature.

The derivation of differential algebraic equations (DAEs) of motion for a constrained N body system with frictionless joints are covered in great detail in [1]. With the constraint manifold denoted by $\boldsymbol{\Phi}$, the equations of motion take the form as in equation (1) for the i^{th} body, where subscript i indicates correspondence to i^{th} body in the system. Here, $\mathbf{M}$ is the mass matrix of the body, and $\mathbf{f}$ is the vector of external forces and torques. The term $\mathbf{C}(\mathbf{q})\boldsymbol{\lambda}$ represents constraint forces acting on the center of mass of the body, where $\mathbf{C}(\mathbf{q})$ is the matrix of coefficients of constraint forces, and $\boldsymbol{\lambda}$ are the corresponding Lagrangian multipliers. Evaluation of $\mathbf{C}(\mathbf{q})$, $\boldsymbol{\Phi}(\mathbf{q})$ and the operator $\Pi'^T(\boldsymbol{\Phi})$ is illustrated in [1].

$$\mathbf{M}_i\,\dot{\mathbf{v}}_i + \mathbf{C}(\mathbf{q})_i\boldsymbol{\lambda}_i = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}})_i, \quad \forall i \in [1,2,\ldots,N] \quad \text{where} \quad \boldsymbol{\Phi}(\mathbf{q})_i = \mathbf{0}, \quad \mathbf{C}(\mathbf{q})_i = \begin{bmatrix} \boldsymbol{\Phi}_{\mathbf{r}}{}^T \\ \Pi'^T(\boldsymbol{\Phi}) \end{bmatrix}_i. \tag{1}$$

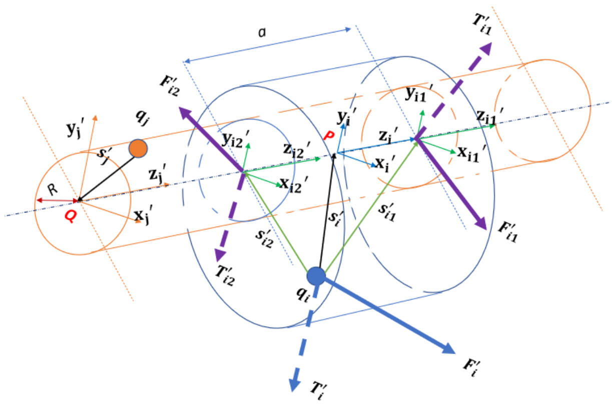

Figure 1: Schematic diagram of contact forces in Cylindrical/Revolute joint

Consider bodies i and j forming a cylindrical (or revolute) pair as shown in Figure 1. The radius of the cylindrical surface is R. The "virtual" points of action in a cylindrical/revolute joint lie on the periphery of the ends of the cylindrical surface in contact, whose centers' locations are given by $\mathbf{s}'_{i1}$ and $\mathbf{s}'_{i2}$ respectively. Since the resulting constraint torques act about the center of mass $\mathbf{q}_i$, the constraint forces can be resolved into contact forces acting at their "virtual" points of action, under dynamic equilibrium about $\mathbf{q}_i$. In local body frame, they form the following system of linear equations:

$$\mathbf{L}\mathbf{f}_c = \mathbf{b} \quad F_{n1} = \|\mathbf{F}'_{i1}\|, \quad F_{n2} = \|\mathbf{F}'_{i2}\| \tag{2a}$$

$$\mathbf{L} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \tilde{\mathbf{s}}'_{i1} & \tilde{\mathbf{s}}'_{i2} \\ \mathbf{s}'^{\,T}_{i1} - \mathbf{s}'^{\,T}_{i2} & \mathbf{0} \\ \mathbf{0} & \mathbf{s}'^{\,T}_{i1} - \mathbf{s}'^{\,T}_{i2} \end{bmatrix} \quad \mathbf{f}_c = \begin{bmatrix} \mathbf{F}'_{i1} \\ \mathbf{F}'_{i2} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{A}_i^T\mathbf{F}'_i \\ \mathbf{T}'_i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_i^T\boldsymbol{\Phi}_{\mathbf{r}_i}^T\boldsymbol{\lambda} \\ \Pi_i'^T(\boldsymbol{\Phi})\boldsymbol{\lambda} \\ 0 \\ 0 \end{bmatrix}. \tag{2b}$$

where $\mathbf{L} \in \mathbb{R}^{8\times 6}$ is a constant matrix of rank 6, and F_{n1} and F_{n2} are the magnitudes of the normal contact forces at the two virtual contact points. The equation (2) can be solved as $\mathbf{f}_c = \mathbf{L}^{+}\mathbf{b}$ where the pseudo-inverse $\mathbf{L}^{+}$ needs to be computed only once for the complete simulation. The corresponding unit direction vectors of contact forces $\mathbf{F}'_{ik}$ for contact points $k \in [1,2]$ in body-fixed frame can be evaluated as $\mathbf{n}'_{ik} = \frac{\mathbf{F}'_{ik}}{\|\mathbf{F}'_{ik}\|}, \mathbf{n}'_{jk} = -\mathbf{n}'_{ik}$. These unit normal vectors can be used to track the "virtual" contact points. In global frame of reference, the location of virtual contact points $\mathbf{c}_1$, and their respective velocities $\dot{\mathbf{c}}_{ik}$ and $\dot{\mathbf{c}}_{jk}$ in bodies i and j:

$$\mathbf{c}_1 = \mathbf{r}_i + \mathbf{A}_i(\mathbf{s}'_{i1} - R\mathbf{n}'_{i1}), \quad \mathbf{c}_2 = \mathbf{r}_i + \mathbf{A}_i(\mathbf{s}'_{i2} - R\mathbf{n}'_{i2}). \tag{3a}$$

$$\dot{\mathbf{c}}_{ik} = \dot{\mathbf{r}}_i + \mathbf{A}_i\tilde{\boldsymbol{\omega}}'_i(\mathbf{s}'_{ik} - R\mathbf{n}'_{ik}), \quad \dot{\mathbf{c}}_{jk} = \dot{\mathbf{r}}_j + \mathbf{A}_j\tilde{\boldsymbol{\omega}}'_j(\mathbf{s}'_{jk} - R\mathbf{n}'_{jk}). \tag{3b}$$

Indeed, the equations (3) differ for prismatic joints based on the cross-section of the prismatic pair. However, the equations (2) provide solution irrespective of the type of joint. With the velocities of virtual points of action, the slip velocities $\mathbf{v}_{sk}$ and their respective direction unit vectors of relative slip velocities $\mathbf{s}^v_{ik}$ and $\mathbf{s}^v_{jk}$, can be calculated.
Friction, opposes the relative motion between two bodies in contact and hence the friction force acts in the direction opposite to that of relative velocity. For a system of N bodies with C virtual action points, let the scalar value of friction force at contact $k \in [1,2,\ldots,C]$ be $F_{fk} = \mu(v_{sk}).F_{nk}$ where $\mu(v_{sk})$ is the coefficient of friction as a function of slip speed. For a system with $N = 2$ and $C = 2$ as shown in Figure 1, the vector $\mathbf{Q}_f \in \mathbb{R}^{6N}$ composed of friction forces and the torques generated by them can be evaluated using the direction matrix $\mathbf{C}_f \in \mathbb{R}^{6N\times C}$ as per the following equations:

$$\mathbf{Q}_f = \begin{bmatrix} -\mathbf{s}^v_{i1} & -\mathbf{s}^v_{i2} \\ -\tilde{\mathbf{c}}'_{i1}\mathbf{A}_i^T\mathbf{s}^v_{i1} & -\tilde{\mathbf{c}}'_{i2}\mathbf{A}_i^T\mathbf{s}^v_{i2} \\ -\mathbf{s}^v_{j1} & -\mathbf{s}^v_{j2} \\ -\tilde{\mathbf{c}}'_{j1}\mathbf{A}_j^T\mathbf{s}^v_{j1} & -\tilde{\mathbf{c}}'_{j2}\mathbf{A}_j^T\mathbf{s}^v_{j2} \end{bmatrix} \begin{bmatrix} \mu(v_{s1}).F_{n1} \\ \mu(v_{s2}).F_{n2} \end{bmatrix} = \mathbf{C}_f(\mathbf{q},\dot{\mathbf{q}})\mathbf{F}_f(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\lambda}), \tag{4}$$

Hence, including friction amends the equation (1) by adding the friction force as follows:

$$\mathbf{M}_i\dot{\mathbf{v}}_i + \mathbf{C}(\mathbf{q})_i\boldsymbol{\lambda}_i + \mathbf{Q}_{fi} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}})_i, \quad \forall i \in [1,2,\ldots,N] \tag{5}$$

Then, the equations (5) can be then discretized using the schemes presented in [1]. The preliminary results are shown in Figure 2 for a pendulum representing a revolute joint. The static and dynamic coefficients of friction were taken as $\mu_s = 0.3$ and $\mu_d = 0.2$ respectively. Energy loss can be observed in the decreasing amplitude in position plots. Stiction was also captured when the direction of rotation changed. The work further aims to study the same for other mechanical joints.

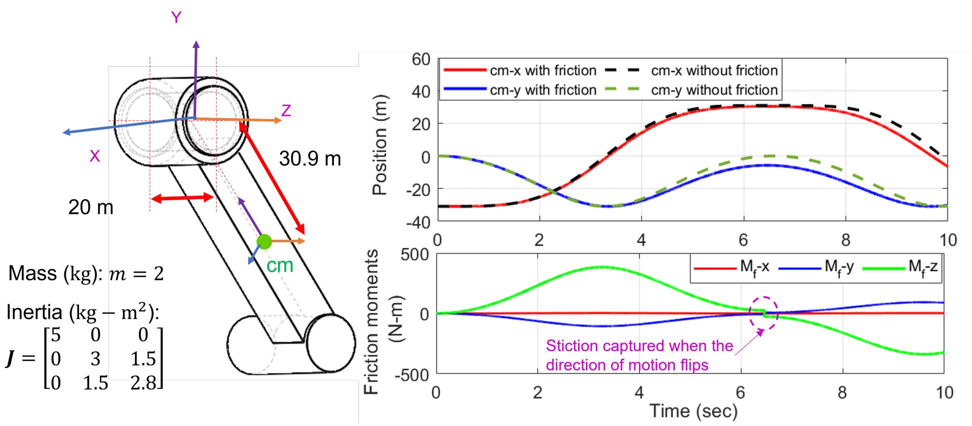

Figure 2: Rigid pendulum: Revolute joint with friction

References

[1] Kissel, A., Negrut, D., Taves, J.: Dwelling on the connection between SO(3) and rotation matrices in rigid multibody dynamics. Part 1: Description of an index-3 DAE solution approach. Proceedings of the ASME IDETC-CIE 2021.

[2] Haug, E.J.: Computer-aided kinematics and dynamics of mechanical systems: Volume II: Modern Methods ResearchGate, www.researchgate.net (2020).