# Type-n Fuzzy Control Systems

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***Abstract*— Zadeh, mamdani and TSK are proposed different fuzzy conditional inferences for "if … then … " to approximate with uncertain information. The Zadeh and Mamdani fuzzy conditional inferences are require prior information for consequent part. Prior information of consequent part does not know in some fuzzy control systems. .The proposed fuzzy conditional inference need not know prior information for consequent part. In this paper, fuzzy conditional inference is proposed for "if … then …" when prior information is not available to consequent part. Type-n fuzzy set is studied for fuzzy conditional inference.

**Keywords— fuzzy logic, fuzzy conditional inference, type-n fuzzy control systems**

## I. INTRODUCTION

There are many theories to approximate uncertain information. Until recently probability theory was the only existing theory to approximate uncertain formation. Zadeh [6] proposed fuzzy logic to approximate uncertain information. The fuzzy theory allows us to represent set membership as a possibility distribution. Fuzzy theory is the most effective than the other theories because fuzzy theory depends on degree of belief rather than likelihood (Probability). The fuzzy conditional propositions are of the type if (president part) then (consequent part). Zadeh [5], Mamdani [1] and TSK [2] proposed different methods of fuzzy conditional inference to approximate uncertain information. The Zadeh and mamdani fuzzy conditional inferences are needed prior information for both president and consequent part. There are some applications like fuzzy control systems do not have prior information to consequent part. The TSK fuzzy conditional inferences are need not know prior information for consequent part but it is difficult to compute

In the following, Zadeh, Mamdani and TSK fuzzy conditional are discussed. Different methods are studied for these methods when consequent part of fuzzy condition is not known. The fuzzy control system is given as example. It is necessary to give brief description of fuzzy logic.

## II. FUZZY LOGIC

Zadeh [13] introduced the concept of a fuzzy set as a model of a vague fact. The use of the fuzzy set theory for control systems is now accepted because it is very convenient and believable.

**Definition**: Given some universe of discourse X, a fuzzy set A of X is defined by its membership function μA taking values on the unit interval[0,1] i.e.μA:→[0,1]

Suppose X is a finite set. The fuzzy set A of X may be represented as

$A= \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots\ldots + \mu_A(x_n)/x_n$

Where "+" is union

For instance,

TALL =0.00/5'0" + 0.08/5'4" + 0.32/5'8"+ 0.50/6'0" + 0.82/6'4"

There is an alternative way to defined fuzzy subset with function and is given by [7]

For example,

$YOUNG = \{ \mu_{YOUNG}(x)/x=1 \quad \text{if } x\epsilon[0,25]$

$=[1+((x-25)2 )]-1 \quad \text{if } x\epsilon[25,100]$

Let A and B be the fuzzy sets, and the operations on fuzzy sets are given below

$AVB=\max(\mu_A(x) , \mu_B(y)\}$ Disjunction

$A\Lambda B=\min(\mu_A(x) , \mu_B(y)\}$ Conjunction

$A'=1- \mu_A(x)$ Negation

$A\rightarrow B=\min \{1, (1- \mu_A(x) +\mu_B(y)\}$ Implication

$AXB=\min \{ \mu_A(x) , \mu_B(y)\}/(x,y)$ Relation

$AoR=\min_x\{ \mu_A(x) , \mu_R(x,y)\}/y$ Composition

Zadeh [6] and Mamdani[1] are proposed different fuzzy conditional inferences

The Zadeh fuzzy condition inference s given by

If ($A_1$ and $A_2$ ….. $A_n$) then y is B =

$\min \{1, (1-\min(\mu_{A1}(x), \mu_{A2}(x), \ldots, \mu_{An}(x)) +\mu_B(y)\}$

The Mamdani fuzzy condition inference s given by

If ($A_1$ and $A_2$ ….. An) then y is B =

$\min(\mu_{A1}(x), \mu_{A2}(x), \ldots, \mu_{An}(x) ,\mu_B(x)\}$

The TSK[2] fuzzy condition inference s given by

If ($A_1$ and $A_2$ ….. An) then y is B

$y=f(x_1, x_2, \ldots, x_n)$

## III. MODIFIED TSK FUZZY CONDITIONAL INFERENCE USING T-NORM

Zadeh [5], Mamdani [1] and TSK [2] are proposed fuzzy conditional inference. Zadeh and Mamdani fuzzy inferences need prior information for consequent part in "if … then …"

if $x_1$ is $A_1$ and $x_2$ is $A_2$ and … and $x_n$ is $A_n$ then y is B

Zadeh fuzzy inference is given

$= \min(1, 1-\min(A_1, A_2 ,\ldots ,A_n )+ B)$

The fuzziness of consequent part must be known when Zadeh fuzzy inference is used.

The Zadeh fuzzy conditional inference is not suitable when consequent part is not known.

Mamdani inference is given by

if $x_1$ is $A_1$ and $x_2$ is $A_2$ and … and $x_n$ is $A_n$ then y is B = $\min(A_1, A_2 ,\ldots ,A_n , B)$

The fuzziness of consequent part must be known when Mamdani fuzzy inference is used.

The Mamdani fuzzy conditional inference is not suitable when consequent part is not known.

The TSK fuzzy conditional inferences are not known prior information for consequent part but it is difficult to compute applications like Control Systems.

Consider TSK fuzzy conditional inference

if $x_1$ is $A_1$ and $x_2$ is $A_2$ and … and $x_n$ is $A_n$ then $y=f(x_1, x_2, \ldots, x_n)$ is B

The fuzzy set B is defined as a function of $A_1$ and $A_2$ and … and $A_n$

The proposed method for TSK fuzzy conditional inference may be defined as using t-norm

If $x_1$ is $A_1$ and/or $A_2$ and/or ,…,and/or $A_n$ then y is $B=f(A_1, A_2, \ldots, A_n)$

If $x_1$ is $A_1$ and $A_2$ and ,…,and $A_n$ then y is $B=\min(\mu_{A1}(x), \mu_{A2}(x), \ldots, \mu_{An}(x))$

If $x_1$ is $A_1$ or $A_2$ and $A_3$ then y is $B=f(A_1, A_2, A_3)$

$=\min(\max(\mu_{A1}(x_1), \mu_{A2}(x_2)), \mu_{A3}(x_3))$

Where t-norm is

$t(a \vee b)=\max(a,b)$

$t(a \wedge b)=\min(a,b)$

if x is A1 and x is A2 then x is B=

$B = 0.2/x_1 + 10.6x_2 + 0.9/x_3 + 0.6x_4 + 0.2/x_5$

The fuzziness may be given for rule as

If BZ is Low (0.6)

and BE is Normal (.7)

Then reduce fan speed

. = min(.6,0.7, 1)

=0.6

## IV. PRESENTATION OF FUZZY SET TYPE-N

The fuzzy set type-2 is a type of fuzzy set in which some additional degree of information is provided

**Definition**: Given some universe of discourse X, a fuzzy set type-3 A of X is defined by its membership function $\mu_A(x)$ taking values on the unit interval[0,1] i.e. $\mu_{\tilde{A}}(x) \rightarrow [0,1]^{[0.1][0,1]}$

Suppose X is a finite set. The fuzzy set A of X may be represented as

$A= \mu_{\tilde{A}1}(x_1)/\tilde{A}1+ \mu_{\tilde{A}2}(x_2)/\tilde{A}2+ \ldots\ldots\ldots\ldots+ \mu_{\tilde{A}n}(x_n)/\tilde{A}n$

Cold= { 0.4/mild , 0.6/moderate, 0.9/severe}

The fuzzy set type-2 may be defined as

**Definition**: The fuzzy set type-2 $\tilde{A}$ is characterized by membership function $\mu_{\tilde{A}}:X \rightarrow [0,]^{[0,1],}$ $x \in X$

Suppose X is a finite set. The fuzzy set $\tilde{A}$ may be new represented by

$\tilde{A}= \int\int \mu_{\tilde{A}}(x)= \sum\sum \mu_{\tilde{A}}(x) = (\mu_{\tilde{A}}(x_{1,})/x_1 + \mu_{\tilde{A}}(x_2)/x_{2} +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_1$

$+ (\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_2 +\ldots+$

$(\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_m$

Where y is type-2 fuzzy set.

$\tilde{A}$ = { $(0.1/x_1+0.2/x_2+0.3/x_3+0.35/x_4+0.4/x_5)$/hot $+(0.4/x_1+0.45/x_2+0.5/x_3+0.55/x_4+0.6/x_5)$/worm} $+(0.7/x_1+0.75/x_2+0.8/x_3+0.85/x_4+0.9/x_5)$/cold }

**Definition**: The fuzzy set type-3 $\tilde{A}$ is characterized by membership function $\mu_{\tilde{A}}:X \rightarrow [0,]^{[0,1],[0,1]}$ $x \in X$

Suppose X is a finite set. The fuzzy set $\tilde{A}$ may be new represented by

$\tilde{A}= \int\int \mu_{\tilde{A}}(x,y,z)/= \sum\sum \mu_{\tilde{A}}(x,y,z) = ((\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_1$

$+ (\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_2 +\ldots+$

$(\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_m )z_1+((\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_1$

$+ (\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_2 +\ldots+$

$(\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/y_m )$

$+(\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/z_1$

$+ ((\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/z_2 +\ldots+$

$(\mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n)/x_n)/)z_k$

Where y is type-2 and z is type-3 fuzzy sets

$\tilde{A}$ =

{ $(0.1/x_1+ 0.2/x_2+0.4/x_3+0.6x_4+1.0/x_5)$/moderate cold$+(0.1/x_1+ 0.2/x_2+0.4/x_3+0.6x_4+1.0/x_5)$/medium cold$+ (0.1/x_1+ 0.2/x_2+0.4/x_3+0.6x_4+1.0/x_5)$/Sevier cold}+{ $(0.3/x_1+0.6/x_2+1.0/x_3+0.0.6/x_4+0.3/x_5)$/ Moderate worm + $(0.3/x_1+0.6/x_2+1.0/x_3+0.0.6/x_4+0.3/x_5)$/ Medium worm $+(0.3/x_1+0.6/x_2+1.0/x_3+0.0.6/x_4+0.3/x_5)$/ Sevier cold }+{ $(0.8/x_1+0.7/x_2+0.6/x_3+0.4/x_4+0.1/x_5)$/Moderate $_{hot}$+ $(0.8/x_1+0.7/x_2+0.6/x_3+0.4/x_4+0.1/x_5)$/medium hot$_+$$(0.8/x_1+0.7/x_2+0.6/x_3+0.4/x_4+0.1/x_5)$/Sevier hot}

**Definition**: The fuzzy set type-n $\tilde{A}$ is characterized by membership function $\mu_{\tilde{A}}:X \rightarrow [0,]^{[0,1][0,1]\ldots[0,1],}$ $x \in X$

Suppose X is a finite set. The fuzzy set $\tilde{A}$ may be new represented by

$\tilde{A}= \int\int \mu_{\tilde{A}}(x,y,z, \ldots)/= \sum\sum \mu_{\tilde{A}}(x,y,z,\ldots)$

Rough set theory is another approach to uncertain information[16]. The uncertain Information may be deal with fuzzy logic.

**Definition**: Given some universe of discourse X, a fuzzy rough set is defined as pair $\{t, \mu_d(t)\}$, where d is domains and membership function $\mu_d(x)$ taking values on the unit interval[0,1] i.e. $\mu_d(t) \rightarrow [0,1]$, where $t_i \in X$ is tuples .

TABLE I. Fuzzy relation

| | $d_1$ | $d_2$ | . | $d_m$ | $\mu$ |
|---|---|---|---|---|---|
| $t_1$ | $a_{11}$ | $a_{12}$ | . | $a_{1m}$ | $\mu_d(t_1)$ |
| $t_2$ | $a_{21}$ | $a_{22}$ | | $A_{2m}$ | $\mu_d(t_2)$ |
| . | . | . | . | . | . |
| $t_n$ | $a_{1n}$ | $a_{1n}$ | . | $A_{nm}$ | $\mu_d(t_n)$ |

Let C and D be the fuzzy rough sets .

The operations on fuzzy rough sets are given as

$1\text{-}C= 1- \mu_C(x)$ Negation

$C \vee D=\max\{\mu_C(x) , \mu_D(x)\}$ Union

$C \wedge D=\min\{\mu_C(x) , \mu_D(x\}$ Intersection

The fuzzy rough set $\tilde{A}$ of fuzzy information system may be represented in terms of fuzzy rough set.

**Definition**: The fuzzy rough set $\tilde{A}$ is characterized by membership function $\mu_{\tilde{A}}:XxY \rightarrow [0,1]$, where $x_i \in X$ is object and $y_j \in Y$ is fuzzy attribute and $\tilde{a}_{ij} \in \tilde{A}$ is fuzziness of type-2

TABLE II. Type-2 Fuzzy relation

| | | $d_1$ | $d_2$ | . | $d_m$ | $\mu$ |
|---|---|---|---|---|---|---|
| $t_1$ | D1 | $\tilde{a}_{11}$ | $\tilde{a}_{12}$ | . | $\tilde{a}_{1m}$ | $\mu_{\check{D}1}(t_1)$ |
| $t_2$ | D2 | $\tilde{a}_{21}$ | $\tilde{a}_{22}$ | | $\tilde{a}_{2m}$ | $\mu_{\check{D}2}(t_2)$ |
| . | | . | . | . | . | . |
| $t_n$ | Dn | $\tilde{a}_{1n}$ | $\tilde{a}_{1n}$ | . | $\tilde{a}_{nm}$ | $\mu_{\check{D}n}(t_n)$ |

Where $\check{D}$ is type 2 fuzzy set

Suppose X is a finite set. The fuzzy rough set $\tilde{A}$ of X may be new represented by

$\tilde{A}=\int\int \mu_{\tilde{A}}(x,y)/x/y= \sum\sum \mu_{\tilde{A}}(x,y) = (\mu_{\tilde{A}}(x_1,y_1)/x_1 + \mu_{\tilde{A}}(x_2,y_1)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n,y_1)/x_n)/y_1$
$+ (\mu_{\tilde{A}}(x_1,y_2)/x_1 + \mu_{\tilde{A}}(x_2,y_2)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n,y_2)/x_n)/y_2 +\ldots+ (\mu_{\tilde{A}}(x_1,y_m)/x_1 + \mu_{\tilde{A}}(x_2,y_m)/x_2 +\ldots+ \mu_{\tilde{A}}(x_n,y_1)/x_n)/y_m$

$\tilde{A}'=1-\mu_{\tilde{A}}(x,y)$

$\tilde{A}= \{ (0.1/x_1+ 0.2/x_2+0.3/x_3+0.35/x_4+0.4/x_5)/y_1 +(0.4/x_1+0.45/x_2+0.5/x_3+0.55/x_4+0.6/x_5)/y_2 +(0.7/x_1+0.75/x_2+0.8/x_3+0.85/x_4+0.9/x_5)/y_3 \}$

TABL III. Type-2 Fuzzy relation example

| | x1 | x2 | x3 | x4 | x5 |
|---|---|---|---|---|---|
| Low | 0.9 | 0.7 | 0.6 | 0.4 | 0.1 |
| Normal | 0.3 | 0.6 | 0.9 | 0.6 | 0.3 |
| High | 0.1 | 0.4 | 0.6 | 0.8 | 0.9 |

Let $\hat{C}$ and $\check{D}$ be the fuzzy rough sets .

The operations on fuzzy rough sets are given as

$\hat{C}V\check{D}=\max\{\mu_{\hat{C}}(x,y) , \mu_{\check{D}}(x,y)\}$ Disjunction

$\hat{C}\Lambda\check{D}=\min\{\mu_{\hat{C}}(x,y) , \mu_{\check{D}}(x,y)\}$ Conjunction

$\hat{C} \rightarrow \check{D} =\min \{1, (1- \mu_{\hat{C}}(x,y) +\mu_{\check{D}}(x,y)\}$ Implication

$\hat{C}x\check{D}=\min\{\mu_{\hat{C}}(x,y) , \mu_{\check{D}}(x,y)\}$ Relation

## V. FUZZY CONTRO: SYSTEMS

Zadeh introduced fuzzy algorithms . The fuzzy algorithm is set of fuzzy statements. The fuzzy conditional statement is defined as fuzzy algorithm

if $x_i$ is $A1_i$ and $x_i$ is $A2_i$ and … and $x_i$ is $An_i$ then $y_i$ is $B_i$

The precedence part may contain and/or/not.

The Fuzzy Control System consist of set of fuzzy rules

If (set of conditions are satisfied then (set of consequences inferred)

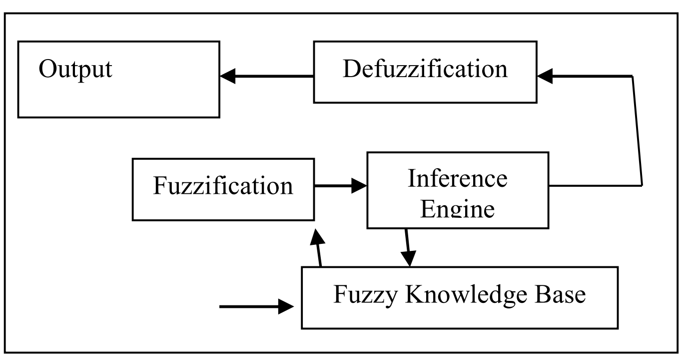


Figure.1 Fuzzy Control System

The Fuzzy control system contains fuzzy variable may be represented in decision table

TABLE III. Fuzzy inference table

| A1 | A2 | .. | An | | B |
|---|---|---|---|---|---|
| A11 | A12 | .. | A1n | | B1 |
| A21 | A22 | .. | A2n | | B2 |
| | | | | | |
| Am1 | Am2 | .. | Amn | | Bmn |

The relational model of fuzzy control system is given as

TABLE IV. Type-2 Fuzzy control system

| Condition | BZ (Burning Zone Temperature) | BE (Back-End Temperature) | Action |
|---|---|---|---|
| AND | Low (0.3) | Low (0.2) | Reduce Klin Speed |
| AND | Low (0.4) | Low (0.3) | Reduce Fuel |
| AND | Low (0.27) | Low (0.35) | Increase Fan Speed |
| AND | Low (0.35) | High (0.9) | Reduce Fuel |
| AND | Low (0.26) | Normal (0.6) | Reduce Fan Speed |

For instance, consider the relational model of type-3 fuzzy control system

TABLE V. Type-3 Fuzzy control system

| Condition | BZ (Burning Zone Temperature) | BE (Back-End Temperature) | Action |
|---|---|---|---|
| AND | Drastically Low (0.2) | Low (0.1) | Reduce Klin Speed |
| AND | Drastically Low (0.3) | Low (0.2) | Reduce Fuel |
| AND | Slightly Low (0.2) | Low (0.7) | Increase Fan Speed |
| AND | Low (0.7) | High (0.65) | Reduce Fuel |
| AND | Low (0.1) | Normal (0.5) | Reduce Fan Speed |

**Defuzzification**

Usually Centroid technique is used for defuzzification. It finds value representing Centre of Gravity(COG) aggregated fuzzy generalized fuzzy set.

$COG=\Sigma C_i \mu_{Ai}(x)/ \Sigma C_i$

For instance,

Speed= { 0.1/20 +0.3/40 + 0.5/60 + 0.7/80 +0.9/100 }

COG=( 0.1*20 +0.3*40 + 0.5*60 + 0.7*80 +0.9*100) / ( 0.1 +0.3 + 0.5 + 0.7 +0.9 )=73.6

## VI. Conclusion

The fuzzy logic deals uncertain information with belief rather than probability. Zadeh , Mamdani and TSK are proposed fuzzy conditional inferences. The Zadeh and mamdani fuzzy conditional inferences are required prior information for consequent part . The TSK fuzzy conditional inferences are need not required prior information for consequent part but it is difficult compute. Some methods are studied for fuzzy conditional inferences when prior information is not available to consequent part. Zadeh method is not suitable when prior information is available to consequent part. Type-n fuzzy control system is discussed The fuzzy Control Systems are given as application.


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