To Manage Fisheries with Complexity and Chaos Theories

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ABSTRACT

Chaos theory - and models related to non-linear dynamic systems - has increased its importance last decades. In fact, chaos is one of the concepts that most rapidly have expanded in what research topics respects. Chaos is ordinarily disorder or confusion; scientifically it represents a disarray connection, but basically it involves much more than that. Change and time are closely linked, and they are essential when considered together as chaos theory foundations are intended to be understood. Given the large number of applications in several areas, the goal of this work is to present chaos theory - and dynamical systems such as the theories of complexity - in terms of the interpretation of ecological phenomena. The theory of chaos applied in the context of ecological systems, especially in the context of fisheries, has allowed the recognition of the relevance of this kind of theories to explain fishing events. It raised new advances in the study of marine systems, contributing to the preservation of fish stocks.

Keywords: Chaos theory, dynamical systems, complex adaptive systems, fisheries.

INTRODUCTION

The Mankind's oldest concern inhabits necessarily on its survival. For thousands of years the Humanity always lived with an almost steady level in the way of living. Production followed the growth of the population. To the generality of the times and to the generality of the people, the historical rule was a life with the strictly necessary for the populations' day-to-day.

The development is an exception in the history of the Humanity that exists just for a period not longer than 250 years. The Humanity had never had a so significant, consistent, and intense transformation in its history. In this period, a complete revolution in the lifestyle of the world-wide population happened to the countries currently appointed as developed countries. However, its effects were not bordered to these zones. Its presence has been extensive to the whole world. In particular, with what it is assigned as external aid, the rich countries changed the stages of development of a great part of the globe, shortening the development stages of many countries and provoking several kinds of unbalances and disturbs - see, for example, what happened with vaccines on the field of health, with the pollution, with all the problems emerged on the feeding area and with other many forms of intrusion, with visible effects and heavy direct impacts.

This development represents a large step for the Humanity. The comfort, the best levels of health and the best standards of life correspond to a set of advantages which medal reverse represents so big costs that their effects we are not able to study yet.

It is not the progress and the development that may be contested but the way the societies do not find ways to get a supported and balanced development, respecting the principles of equity that must exist among our specie and the others.

Throughout the 20th century the extreme exploitation of resources brought out very big concerns about their preservation. Along the time the immoderation has shown the necessity of measures to avoid the depletion of many resources. That is why common resources become studied worldwide. Nowadays, they are an important subject to analyze resources problems. In the last few decades, international organizations have successively looked for balances to the problem of fragile resources preservation and environmental questions, searching for solutions. The international agents and the private interests are numerous, and the convergences seem to be traditionally very difficult. National and international organizations and States have seen the balances of life and resources on Earth to be frequently broken.

It is based on the existence of this type of position divergences that we intend to analyze the framings and this idea relative to the Earth resources. Our study intends to conjugate the need of resources preservation - specially the common resources - with the perception of the way the involved agents face the subjects studied.

We intend to answer to the necessity of finding solutions that aim the preservation and the improvement of the conditions of life in society compatible with the environment.

The existing risks to the Humanity and to all the live resources require that some theoretical contexts are posed in a way that allow investigators to develop possible solutions and to find out the best ways to achieve them.

Since its beginning, chaos has become rapidly a developing field and many of the progresses in this area were revealed just since the 1970s. This means that many facets of chaos are distant from being understood or determined yet, making of it a field with great potential. It is important to note that nowadays chaos is extremely difficult to be identified in real world information in order to be workable. It is possible to find it in mathematical computer problems to be solved and laboratory research. As soon as the idea of nonlinearity[1] is introduced into theoretical models, chaos gets obvious. A very complex structure is observed in the field data. Simple patterns can be found and approximated; complex patterns are another matter. In any event, we can't just grab a nice little set of data, apply a simple test or two, and declare "chaos" or "no chaos." (Williams, 1997).

The chaos theory involves multiple interactions and supposes the existence of an enormous number of interrelations, with direct developments in vast fields of study. It got an important role in the context of recent theoretical developments of the non-equilibrium theories. The word chaos assumes the idea of the existence of turbulence and disorder; an unwanted chance or even the idea of an "abyss". The predisposition to a profound change in the direction of a phenomenon generates an own force, understood as a depth change that results from small changes in their initial conditions. The chaos is, from this point of view, something extremely sensitive to the initial conditions. It is interesting to note, however, that the chaotic system normally seems to develop itself in a very smooth and orderly way, although inside changes may be complex and paradoxical.

Recent developments in the dynamical systems theories, which require the existence of an inherent complexity of the systems themselves that are based on a set of large inside interactions, led in some cases to understand and highlight self-organizing systems, revealing strong strengths and reinforcing their internal cohesion factors.

Given the non-linearity conditions of the nature phenomena, theories which are based on the dynamics of non-equilibrium seem to explain quite well the spatial and temporal heterogeneity observed in ecological systems. Chaos occurs often in deterministic, nonlinear, dynamical systems.

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[1] Nonlinear means that output isn't directly proportional to input, or that a change in one variable doesn't produce a proportional change or reaction in the related variable(s).

The disturbances and heterogeneity are interdependent factors that create opportunities for re-colonization and determine the structure of communities. It is important to note, for reflection about the effects of the disturbances in the systems, that the ability to recover the ecological systems depends, to some extent, on the existence of refuge areas both for flora and fauna, acting as reservoirs of re-colonizers, after the disturbances occurred in the ecosystem.

Many live resources, particularly many marine resources have suffered drastic reductions motivated by their overexploitation. Populations of many species have been led to the rupture and close to the extinction. How can Humanity modify this state of things? Is there a line of evolution that helps to explain this kind of events? How can we shape these facts in this perspective? Is there any ways to invert these trends? Or simply to find the principles that underline the facts? The main agents in this process may be questioned? The supranational institutions just emit simple indicative rules.

CHAOS IN DYNAMICAL SYSTEMS' THEORIES

A dynamical system represents moving, changing, or evolving in time. For this reason, chaos deals with dynamical systems theory - the study of phenomena that vary with time - or nonlinear dynamics - the study of nonlinear movement or evolution.

Dynamical systems fall into one of two categories, depending on whether the system loses energy. A conservative dynamical system has no friction and it doesn't lose energy over time. In contrast, a dissipative dynamical system has friction; it loses energy over time and therefore always approaches some asymptotic or limiting condition. That asymptotic or limiting state, under certain conditions, is where chaos occurs (see Williams, 1997).

The dynamical systems theories have been applied to numerous areas of knowledge. In the 80's, several exact sciences - physics, chemistry or biology, for example - and some social sciences - economics or management or even the sociology - still had their own objects of study and their own methods of analysis and each one of them was different from the others. The science has been branched and specialized, so that each one uses to have its own world. Recently new forms of analysis, looking for an integrated study, have emerged (Filipe, 2006; Filipe *et al*, 2007).

The theory of chaos and complexity theory itself reflect the phenomena that in many activities, such as fisheries, are translated into dynamic forms of analysis and reflect a very complex and widespread reality, specific of complex systems. That reality falls within a range of situations integrated in a broader context, which is expressed in the theory itself but also in terms of their

own realities - fisheries, for example - dynamic, complex, and often chaotic features in their essence.

The theory of chaos stresses that the world does not necessarily work as a linear relationship with perfectly defined or with direct relations in terms of expected proportions between causes and effects. The chaos occurs when a system is very sensitive to the initial conditions. These initial conditions are the measured values for a given initial time. The presence of chaotic systems in nature seems to place a limit on our ability to apply physical deterministic laws to predict movements with any degree of certainty. Indeed, one of the most interesting subjects in the study of chaotic systems is the question of whether the presence of chaos may or may not produce ordered structures and patterns on a wider scale. In the past, the dynamic systems showed up completely unpredictable and the only ones who could aspire to be understood were those that were represented by linear relationships, which are not the rule. On the contrary, there are some situations clearly isolated.

Today, with the help of computers, it is possible to make extremely complex calculations and to understand better the occurrence of chaos.

As Williams (1997) says, phenomena happen over time either as at discrete, separate or distinct, intervals[2] or as continuously[3]. Discrete intervals can be spaced evenly in time or irregularly in time. Continuous phenomena might be measured continuously. However, we can measure them at discrete intervals[4]. Special types of equations apply to each of those two ways in which phenomena happen over time. Equations for discrete time changes are difference equations and are solved either analytically or by iteration methods. In contrast, equations based on a continuous change - continuous measurements - are differential equations. The term "flow" often is associated to differential equations[5].

Differential equations are often the most accurate mathematical way to describe a smooth continuous evolution. However, some of these equations are difficult or impossible to solve. In contrast, difference equations usually can be solved right away. Furthermore, they are often acceptable approximations of differential equations. Olsen and Degn (1985) say that difference equations are the most powerful vehicle to the understanding of chaos.

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[2] Examples are the of occurrence earthquakes, rainstorms or volcanic eruptions.
[3] Examples are air temperature and humidity or the flow of water in perennial rivers.
[4] For example, we may measure air temperature only once per hour, over many days or years.
[5] To some authors (see Bergé *et al.,* 1984), a flow is a system of differential equations. To others (see Rasband, 1990), a flow is the solution of differential equations.

Many scientists see, with particular interest, the theory of chaos as a way to explain the environment. Therefore, the theory of chaos stresses the fundamental laws of nature and natural processes and requires a course for a constant evolution and recreation of nature. The theory of chaos allows to realize the endless alternative ways leading to a new form or new ways that will be disclosed and that eventually emerge from the chaos as a new structure. The reality is a process in which structure and chaos rotate between form and deformation in an eternal cycle of death and renewal. Conditions of instability seem to be the rule and, in fact, a small inaccuracy in the conditions of departure tends to grow to a huge scale. Basically, two insignificant changes in the initial conditions for the same system tend to end in two situations completely different. This situation is known, poetically, as the "butterfly wing effect". A small movement of the wings of a butterfly can have huge consequences. It is the microscopic turbulence having effects in a macroscopic scale - an effect called by Grabinski (2004) as hydrodynamics. Mathematically, the "butterfly wing effect" corresponds to the effect of chaos, which can be expressed as follows.

Given the initial conditions:

$$x_1, x_2, x_3, ..., x_n$$

it is possible to calculate the final condition given by

$$FinalResult = f(x_1, x_2, x_3, ..., x_n)$$

If the initial conditions x_i have a margin of error (variation), the result will be influenced by the existence of this margin. If these margins in x_i are small as the margin of error in the result, we have a situation non-chaotic. Otherwise, if the margins of error in x_i are small but the result has a big variation, there is a chaotic situation. Therefore, small variations in initial conditions can lead to a major effect in the outcome. Sometimes small changes in x_i have exponential effects on the result due to the passage of time.

This effect can be demonstrated mathematically[6] using the Lyapunov Exponent[7] (see Grabinski, 2008). Given the initial value x_0 and ε being its - arbitrarily small - variation, we are conducted

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[6] Several statistics may indicate chaos and can express how chaotic a system is. One of the most important statistics to measure magnitude of chaos is at present Lyapunov exponents. Other statistics could be presented as the Kolmogorov-Sinai entropy or the mutual information or redundancy.

[7] A Lyapunov Exponent is a number that reflects the rate of divergence or convergence, averaged over the entire attractor, of two neighbouring phase space trajectories. Trajectory divergence or convergence has to follow an exponential law, for the exponent to be definable.

to an initial value between x_0 and $x_0 + \varepsilon$. The general form of Lyapunov indicator is presented by

$$x_{n+1} = f(x_n)$$

that after N iterations lead to a value for x_n between:

$$f^N(x_0) \qquad \text{and} \qquad f^N(x_{0+\varepsilon})$$

being the difference between these two values

$$f^N(x_{0+\varepsilon}) - f^N(x_0) \; \Box \; \varepsilon e^{N\lambda(x_0)}$$

where λ is a parameter.

Dividing both sides by the variation ε and assuming the limit $\varepsilon \to 0$, we have a differential quotient. Making its logarithm and assuming the limit $N \to \infty$, we get the final definition of the Lyapunov Exponent

$$\lambda(x_0) = \lim_{N \to \infty} \frac{1}{N} log \mid df^N(x_0)/dx_0 \mid$$

and there is chaos when $\lambda > 0$.

Through this function the chaos exists when arbitrary small variations in initial conditions grow exponentially with a positive exponent.

Grabinski also says that the nonlinearity is the main characteristic of a chaotic situation. Mathematically, the nonlinear functions to be considered chaotic should be based on variables with some resistance. The author also argues that is not enough to describe the chaotic situations, such as turbulence, but it is necessary to find ways to better cope with the nonlinearity. A smooth flow of a river, non-chaotic, that can be described in quantities like the flow velocity can reach a chaotic behavior with variations of many situations. The best example is a waterfall where the speed of the flow reaches a certain point. In a smoothly flowing river, it is easy to calculate or predict the flow velocity of the river at any point. However, to calculate it in a river with a waterfall, it is necessary to introduce chaos. To make this calculation, man has focused on the construction of super computers that have shown to be useless due the infinity of factors that may cause turbulence in the flow of the river. Thus, the analysis of frequency on the change of flow's velocity is much more promising than the analysis of velocities themselves.

Moreover, Grabinski, shows the situation in which there is chaos on a microscopic scale but not on a macroscopic scale - the hydrodynamics. An example is a glass of water resting on a table, a not chaotic event. A slight disturbance on the table causes a small flow on a macroscopic level in the water. However, a microscopic observation reveals a great agitation of millions of molecules, a chaotic event. This is a situation where there is chaos on the microscopic scale but a smooth flow on the macroscopic scale. Mathematically Grabinski presents hydrodynamics equations which combine the chaos theory with business situations. For that, he presents the function about the value of a company (v) that depends on two variables, the revenue (r) and the number of employees (n):

$$v(r,n) = v_0 + a_{10}r + a_{01}n + a_{11}rn + a_{20}r^2 + a_{02}n^2 + \dots$$

being a_{ij} general parameters. For $n = 0$ (no employees) or $r = 0$ (no revenue) the company doesn't exist because the function value is equal to 0. So some terms of the function must be removed $(v_0 = a_{10} = a_{01} = a_{20} = a_{02} = \dots = 0)$. The general form comes as

$$v(r,n) = a_{11}rn + a_{21}r^2n + a_{12}r n^2 + a_{22}r^2n^2 + \dots$$

Now to the case of symmetry, r and n could be negative. A negative employee means that the employee is paying to work and negative revenue means that the company is paying the costumer to consume. So the previous formula can lead to negative results if r and n change signs simultaneously. Only these terms are allowed for which the sums of the powers of r and v are even numbers. Thus, the general expression of the equation is

$$v(r,n) = a_{11}rn + a_{22}r^2n^2 + \dots$$

CHAOS, DYNAMICAL SYSTEMS' THEORY AND ECOLOGY

Chaos, that is, deterministic chaos, deals with long-term evolution - how something changes over a long time. A chaotic time series looks irregular. Two of chaos's important practical implications are that long-term predictions under chaotic conditions are worthless and complex behaviour can have simple causes. Chaos is difficult to identify in real world data because the available tools generally were developed for idealistic conditions that are difficult to fulfil in practice (see Williams, 1997).

The ecology where many things are random and uncertain, in which everything interacts with everything at the same time is, itself, a fertile area for a cross search to the world explanations (Filipe *et al*, 2005).

Lansing (2003) states that the initial phase of the research of nonlinear systems was based on the deterministic chaos, and it was later redirected to new outbreaks of research focusing on the systems properties, which are self-organizing. What is called anti-chaos. It also says that the study of complex adaptive systems, discussed in the context of non-linear dynamic systems, has become a major focus of interest resulting from the interdisciplinary research in the social sciences and the natural sciences.

The theory of systems in general represents the natural world as a series of reservoirs and streams governed by various feedback processes. However, the mathematical representations were ignoring the role of these adjustment processes.

The theory of complex adaptive systems part of the theory of systems, although it has in specific account the diversity and heterogeneity of systems rather than representing them only by reservoirs. It explicitly considers the role of adaptation on the control of the dynamics and of the responses of these heterogeneous reservoirs. This theory allows ecologists to analyze the reasons inherent to the process at the lower levels of the organization that lead to patterns at higher levels of organization and ecosystems. The adaptive systems represent one of the means to understand how the organization is produced to a large scale and how it is controlled by processes that operate at lower levels of organization. According to Lansing (2003), came to be a general idea involving physical and mathematical complexity that is hidden behind systems very simple.

Considering a system composed by many interactive parts, if it is sufficiently complex, it may not be practical or even not be possible to know the details of each interaction place. Moreover, the interactions can generate local non-linear effects that often it becomes impossible to find a solution even for simple systems. However, diverting us from causal forces that move the individual elements, if we focus on the system behavior, we can highlight certain global behavior standards. However, these behavior standards may hide an associated cost: it cannot be expected to understand the causes at the level of individual behavior.

Indeed, the systems do not match the simple decomposition of the whole into parts and therefore do not correspond to the mere sum of the parts, as living systems are not the juxtaposition of molecules and atoms. Since the molecule to the biosphere, the whole is organized, and each level of integration leads to properties that cannot be analyzed only from mechanisms that have

explanatory value in the lower levels of integration. This corresponds to the appearance of new features to the level of the set that does not exist at the level of the constituent elements. Lansing (2003) believes that the adoption in the social sciences of the idea that complex global patterns can emerge with new properties from local interactions had a huge impact here.

The ecological systems are comparable to systems self-organized as they are open systems which arise far from thermodynamic equilibrium. On self-organized and self-regulated systems, the reciprocal interactions within the system between the structures and the processes contribute to the regulation of its dynamics and the maintenance of its organization; partly due to the phenomena of feedback (see Lévêque, 2002). These systems seem to develop themselves in accordance with the properties referred to the anti-chaotic systems. Indeed, we have auto-regulated systems that channel different initial conditions for the same stage, instead of what is happening with chaotic systems, which are very sensitive to initial conditions (see Kauffman, 1993). These systems would be relatively robust for a particular type of disturbance, to which the components of the system fit, creating a meta-stability that depends not only on the internal interactions within the system but also on external forces that can regulate and strengthen the internal factors of cohesion (see Lévêque, 2002).

Scoones (1999) argues that should be concluded a new commitment in research on the ecological new thinking and he develops its search precisely in ecology around the concepts of chaotic dynamics and systems of non-equilibrium. In turn, Levin (2003) shows that in the study of complex adaptive systems anti-chaos involves the understanding of how the cooperation, alliances and networks of interactions emerge from individual behaviors and how it generates a feed-back effect to influence these behaviors within the spontaneous order and self-organization of ecosystems.

DYNAMICAL SYSTEMS AND CHAOS THEORY AND FISHERIES

To frame some methodological developments, it must be mentioned, first of all, that some characteristics associated with some species support strategic survival features that are exploited by the present theory. Its aim is to find the reasons and the way in which these strategies are developed and the resulting consequences. The species use their biological characteristics resulting from evolutionary ancient processes to establish defense strategies.

However, given the emergence of new forms of predation, species got weaker because they are not prepared with mechanisms for effective protection for such situations. In fisheries there is a predator, man, with new fishing technologies who can completely destabilize the ecosystem. By

using certain fisheries technologies, such as networks of siege, allowing the capture of all individuals of the population who are in a particular area of fishing, the fishers cause the breakdown of certain species, particularly the pelagic ones, normally designated by schooling species.

 To that extent, with small changes in ecosystems, this may cause the complete deterioration of stocks and the final collapse of ecosystems, which in extreme cases can lead to extinction. These species are concentrated in high density areas in small space. These are species that tend to live in large schools.

Usually, large schools allow the protection against large predators. The mathematical theory, which examines the relationship between schools and predators, due to Brock and Riffenburgh (see Clark, 1974), indicates that the effectiveness of predators is a reverse function of the size of the school. Since the number of fish that a predator can consume has a maximum average value, overcoming this limit, the growth of school means a reduction in the rate of consumption by the predator. Other aspects defensive for the school such as intimidation or confusing predators are also evidence of greater effectiveness of schools.

However, this type of behavior has allowed the development of very effective fishing techniques. With modern equipment for detecting schools - sonar, satellites, etc. - and with modern artificial fibers' networks - strong, easy to handle and quick placement - fishing can keep up advantageous for small stocks (Bjorndal, 1987; Mangel and Clark, 1983). As soon as schools become scarce, stocks become less protected. Moreover, the existence of these modern techniques prevents an effect of stock in the costs of businesses, as opposed to the so-called search fisheries, for which a fishery involves an action of demand and slow detection. Therefore, the existence of larger populations is essential for fishermen because it reduces the cost of their detection (Neher, 1990). But, the easy detection by new technologies means that the costs are no longer sensitive to the size of the stock (Bjorndal and Conrad, 1987).

This can be extremely dangerous due to poor biotic potential of the species subject to this kind of pressure. The reproductive capacity requires a minimum value below which the extinction is inevitable. Since the efficiency of the school is proportional to its size, the losses due to the effects of predation are relatively high for low levels of stocks. This implies non-feedback in the relation stock-recruitment, which causes a break in the curves of income-effort, so that an infinitesimal increase on fishing effort leads to an unstable condition that can lead to its extinction.

Considering however the fishing as a broader issue, we may consider the modeling of the stocks of fish based on an approach associated with the theory of chaos instead considering the usual prospect based on classical models. Indeed, the issue can be placed within this framework from two different prisms: the traditional vision and the vision resulting from theories of non-equilibrium. Around the traditional Newtonian view, the facts can be modeled in terms of linear relationships: involving the definition of parameters, identifying relevant variables and using differential equations to describe the processes that change slowly over time. For a given system, it should then carry out measurements in a context that remains stable during various periods. Moreover, we may have models based on the theory of chaos. These models are based on non-linear relationships and are very close to several disciplines, particularly in the branch of mathematics that study the invariant processes of scale, the fractals, and in a huge range of other subjects in self-spontaneous creation of order: the theory of disasters or complex systems, for example.

The first way is largely used by most biologists, economists and environmentalists, scientists and technical experts that conduct studies in marine search and senior technicians from state and transnational agencies in the area of fisheries. It treats nature as a system, which has a regular order. But today there are many responsible for fisheries management who also base their decisions on models of chaos.

The classical models center on a particular system and depend on a local analysis, studying several species, age, class, sub-regions of the marine eco-niche, the various ports and their discharges, depending on the account of an even wider range of other factors. Probably, the classic expression of linearity on the dynamics of the population - the principle that nature is orderly, balanced and that has a dynamic balance - is due to Maynard Smith (1968), which argues that the populations either remain relatively constant or regularly vary around an alleged point of balance. In the specific case of commercial fisheries, biologists believe that the fishing effort is often relevant to explain the deviations of actual populations' values for the model. They say that, specially based on studies made in the last decade, fish stocks sustainability should be ensured by the control made through fisheries regulation.

Moreover, some people see nature as not casual and unpredictable. The natural processes are complex and dynamic, and the causal relations and sequential patterns may extend so much in time that may seem to be non-periodical. The data appear as selected random works, disorderly, not causal in their connections and chaotic. The vision provided by nature leads to consider the fish stocks, time, the market and the various processes of fisheries management as likely to be

continuously in imbalance rather than behave in a linear fashion and in a constant search for internal balance. It is this perspective that opens the way for the adoption of the theory of chaos in fisheries. However, the models of chaos do not deny, for themselves, some of the linearity resulting from the application of usual bionomic models. What is considered is that there are no conditions to implement all significant variables in a predictive model. Moreover, in finding that a slight change in initial conditions caused by a component of the system may cause major changes and deep consequences in the system itself. So, the application of the theory of chaos to fishing is considered essential, by many researchers. The theory of chaos depends on a multitude of factors, all major, and in the prospect of this theory all very important at the outset, on the basis of the wide range of unpredictable effects that they can cause.

MODDELING FISHERIES' DISSIPATIVE EFFECT

In order to frame some additional methodological developments, it must be mentioned that some characteristics associated with some species support strategic survival features that are exploited by these species. Considering that, it is important to find the reasons and the way these strategies are developed and the resulting consequences. The species use their biological characteristics resulting from evolutionary ancient processes to establish defense strategies.

However, given the emergence of new forms of predation, species got weaker because they are not prepared with mechanisms for effective protection for such situations. In fisheries there is a predator, man, with new fishing technologies who can completely destabilize the ecosystem. By using certain fisheries technologies, such as networks of siege, allowing the capture of all individuals of the population who are in a particular area of fishing, the fishers cause the breakdown of certain species, particularly the pelagic ones, normally designated by schooling species.

 To that extent, with small changes in ecosystems, this may cause the complete deterioration of stocks and the final collapse of ecosystems, which in extreme cases can lead to extinction. These species are concentrated in high density areas in small space. These are species that tend to live in large schools.

For the application of a mathematical model to fisheries, let's inspire in the model presented in Berliner (1992) related to dissipative systems in the presence of chaos.

In Berliner (1992) it is referred that non invertibility is required to observe chaos for one-dimensional dynamic systems. Additionally, it is said that "everywhere invertible maps in two or more dimensions can exhibit chaotic behavior". The study of strange attractors shows that in the

long term, as time proceeds, the trajectories of systems may become trapped in certain bounded regions of the state space of the system.

The model presented in Berliner (1992) is an example in two dimensions of the Hénon map (displaying the property of having a strange attractor).

The Hénon map appears represented by the equations:

$$x_{t+1} = 1 + y_t - ax_t^2$$

and

$$y_{t+1} = bx_t,$$

for fixed values of a and b and $t = 0,1,\ldots$

This invertible map possesses strange attractors and simultaneously has strong sensitivity to initial conditions.

The Hénon map, representing a transformation from the Cartesian plane to itself, has Jacobian equal to $-b$.

If $0 < b < 1$, the Hénon map contracts the domains to which it is applied. These maps are said to be dissipative (on the contrary, maps that maintain the application domain are said to be conservative).

Presented the model, it is possible now to suggest a model on this basis for fisheries.

So, if a general situation is considered, the following equations may represent a system in which fish stocks, at time t, are given by x_t and catches by y_t. The model is as follows:

$$x_{t+1} = F(x_t) - y_t \text{ and}$$

$$y_{t+1} = bx_t.$$

It is a generalization of Hénon model. The Jacobian is equal to b. As y_{t+1} is a portion of x_t, $0 < b < 1$. So, it is a dissipative model and the values of x_t are restricted to a bounded domain.

Considering the case below:

$$x_{t+1} = x_t - y_t, \text{ and}$$

$$y_{t+1} = bx_t.$$

So,

$$x_{t+2} = x_{t+1} - y_{t+1}$$

and

$$x_{t+2} - x_{t+1} + bx_t = 0.$$

Now, after solving the characteristic equation associated to the difference equation (see Ferreira and Menezes, 1992) it is obtained:

$$k = \frac{1 + \sqrt{1 - 4b}}{2} \text{ or } k = \frac{1 - \sqrt{1 - 4b}}{2}; \quad \text{calling } \Delta = 1 - 4b \text{ and being } 0 < b < 1, \text{ comes that}$$

$$-3 < \Delta < 1.$$

So, $0 < \Delta < 1$ if $0 < b < \frac{1}{4}$ and $-3 < \Delta < 0$ if $\frac{1}{4} < b < 1$, being $\Delta = 0$ when $b = \frac{1}{4}$.

Consequently for $0 < b < \frac{1}{4}$,

$$x_t = A_1 \left(\frac{1 + \sqrt{1 - 4b}}{2} \right)^t + A_2 \left(\frac{1 - \sqrt{1 - 4b}}{2} \right)^t$$

And for $b = \frac{1}{4}$,

$$x_t = \left(A_1 + A_2 t \right) \left(\frac{1}{2} \right)^t$$

Finally, for $\frac{1}{4} < b < 1$

$$x_t = \left(\sqrt{b} \right)^t \left[A_1 \cos \left(\left(\arccos \frac{1}{2\sqrt{b}} \right) t \right) + A_2 \, sen \left(\left(\arccos \frac{1}{2\sqrt{b}} \right) t \right) \right]$$

In these solutions, A_1 and A_2 are real constants.

Note that the bases of t powers are always between 0 and 1. So, $\lim_{t \to \infty} x_t = 0$ and whatever the value of b, the dissipative effect is real, even leading to the extinction of the specie. Of course, this is evident according to the hypotheses of this situation of the model.

Concluding this approach, the general model does not allow to obtain in general such explicit solutions by analytical means. But, of course, with simple computational tools it is possible to obtain recursively concrete time series solutions after establishing the initial value x_0 and to check the dissipative effect.

CONCLUDING REMARKS

The models of chaos are based on non-linear relationships and are very close to several disciplines, particularly in the branch of mathematics that study the invariant processes of scale or the fractals, for example.

Chaos theory got its own space among sciences and has become itself an outstanding science. However there is much left to be discovered. Anyway, many scientists consider that chaos theory is one of the most important developed sciences on the twentieth century.

Aspects of chaos are shown up everywhere around the world and chaos theory has changed the direction of science, studying chaotic systems and the way they work.

We cannot say yet if chaos theory may give us solutions to problems that are posed by complex systems. Nevertheless, understanding the way chaos discusses the characteristics of complexity and analyzes open and closed systems and structures is an important matter of present discussion.

Considering the fisheries in a broad context, the modeling of stocks of fish may be considered on the basis of an approach associated with the theory of chaos instead considering the usual prospect based on classical models.

On the fisheries analysis, overfishing may cause a problem of irreversibility in the recovering of several species, after certain stages for the stocks. Anyway, to analyze the specific situation of each case, it is necessary to obtain enough data to analyze the kind of function which is specific for that particular case, and it must be analyzed the situation for certain phases of fishing, and it must be seen the consequences for these species.

In this paper, a general model that evidences the dissipative effect of catches on fisheries was proposed. Moreover, a particular case, with a system of two difference equations analytically treatable, shows how fishing stocks are dissipated and may tend to the extinction.

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