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Sensitivity of an optical sensor based on capillary microresonator for the measurement the hydrostatic pressure in microfluidics

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Abstract. In this investigation, we report the study of an optical device for the measurement of hydrostatic pressure in fluids. The device studied is a sensor based on a dielectric optical resonator in the form of a capillary that confines the light in its interior through the phenomenon of total internal reflection. In the analytical study of the sensor, the excitation of the azimuthal modes WGMs inside the resonant cavity is considered, so that their sensitivity to changes in the hydrostatic pressure was analyzed as a function of the displacement of wavelengths of resonances in the cavity.

1. Introduction

Optical microresonators are optical devices that are indispensable in circuits of integrated optics, which have applications in different areas of science and engineering. These optical microresonators have been manufactured with different morphologies in the form of capillaries [1-4], discs [5], rings [6-8], toroids [9], spheres [10], bubbles [11], bottles [12], etc, using different materials such as silicon, silica, polymers, with applications in areas such as biology [13], medicine [4], physics [14], chemistry [10] and specifically in the area of sensors for the measurement of temperature [15], humidity [12], refractive index [16] and some other physical variables of interest [6]. Specifically, the cylindrical optical microcavities that have been made using different types of materials, experience a series of resonances commonly known as Whispering Gallery modes WGMs, which are characteristic of the cavities that have rotational symmetry and can be explained through the phenomenon of total internal reflection occurring within these cavities when they are excited through an external source. Recently a series of studies of microcapillary resonators have been reported by its facility to interact simultaneously with two fluids of a different refractive index since they allow the flow of a substance in its interior with the facility. For the case of silica and PMMA polymer microcapillaries, a Q factor of the order of 1×10^4 has been reported [1,17]. In this research, we have analyzed the sensitivity of a capillary resonator for the measurement of hydrostatic pressure of microfluidics, analyzing the propagation of WGMs modes and the spectral shifts in the resonance wavelengths of the cavity in response to changes in hydrostatic pressure within the resonator.

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2. Materials and methods

To perform the theoretical study of the sensitivity of a capillary microresonator for the measurement of hydrostatic pressure in microfluidics, the cavity is considered as a system composed of three layers with different refractive indexes and the WGMs modes are excited in the intermediate layer to the interior of the structure because in that region the light experiences the phenomenon of total internal reflection between the inner and outer interface of the microcavity. For the theoretical analysis, it is possible to determine the wave equation in the microcapillary and in this way it is possible to obtain the wavelengths of resonances and the field distribution of the corresponding WGMs modes with different radial and azimuthal orders. For this, it is necessary to choose a cylindrical symmetry as shown in figure 1.

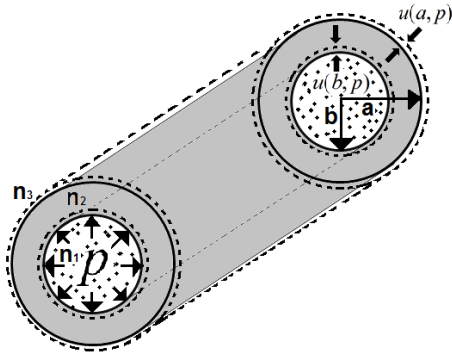


Figure 1. Dielectric capillary scheme. Lateral image of the capillary and Cross-sectional view of the capillary n_1 , n_2 , n_3 , constitute respectively the indices of refraction in the media 1, 2 and 3, while a represent the external radius y b is the internal radius. Outline of the elasto-optical effect inside the capillary by the action of the internal pressurization.

In the cavity, the confinement of the field is given in the material with an index of refraction n_2 which constitutes the wall of the capillary and to ensure the phenomenon of internal total reflection must be satisfied that $n_2 > n_3$ for the case of a cylindrical microcavity with thin wall thickness.

3. Results and Discussion

In the analysis of the polarization of TE modes, the electric field is axial to the axis of the capillary and in this case, we can separate variables for the field as a function of the radial and azimuthal component $E_z(r, \phi) = E_z(r) \exp(\pm il\phi)$, where r, ϕ, z represent the radial, azimuthal and axial coordinates respectively and l is an integer constant called the azimuth order number. According to Maxwell's equations, the resonant modes TE of the cylindrical dielectric cavity satisfy the differential equation in cylindrical coordinates

$$\nabla^2 E_z(r, \phi) + \frac{\partial}{\partial z} \left[E_z(r, \phi) \frac{1}{\varepsilon} \frac{\partial \varepsilon(r)}{\partial z} \right] + k_0^2 n^2 E_z(r, \phi) = 0 \quad (1)$$

Where ε is the electrical permittivity of the material in which the field is propagating, k_0 is the wave number in the vacuum, n is the refractive index and E_z is the electric field polarized along the z-axis. In the case of a composite material with different layers of different refractive index, the refractive index n is n_1 when $r \leq b$, n_2 when $b < r \leq a$ and n_3 when $r > a$, the wall thickness of the capillary is $g = a - b$. The solution for the field given by equation 1 can be expressed as

$$\begin{aligned} E_{z1} &= A_1 J_l(n_1 k_0 r) \\ E_{z2} &= B_1 J_l(n_2 k_0 r) + C_1 Y_l(n_2 k_0 r) \\ E_{z3} &= D_1 H_l^{(1)}(n_3 k_0 r) \end{aligned} \quad (2)$$

Where A_l, B_l, C_l, D_l are complex constants, $J_l(R)$ and $Y_l(R)$ are respectively the cylindrical Bessel functions of first and second type with order l , the function $H_l^{(1)}(R) = J_l(R) + iY_l(R)$ is the first type Hankel function. The solution of the wave equation is expressed in general as $E_z = E_{zi} \exp(il\phi)$ with $E_{zi} := E_{z1}, E_{z2}, E_{z3}$. For the study of TE modes, it is necessary to guarantee the continuity conditions of the field at the boundary at each dielectric interface, therefore some components of the electric and magnetic fields must be continuous. With the application of boundary conditions, it is possible to obtain the eigenvalue equation to determine the resonance wavelengths in the cylindrical cavity for TE modes as a function of the refractive indexes of the medium and the internal and external diameter of the microcavity, which was deduced and its mathematical expression is observed in equation 4.

$$\frac{J_1'(n_1 k_o b) Y_1(n_2 k_o a) - Y_1'(n_2 k_o b) J_1(n_1 k_o b)}{J_1'(n_2 k_o b) J_1(n_1 k_o b) - J_1(n_2 k_o b) J_1'(n_1 k_o b)} = \frac{Y_1'(n_2 k_o a) H_1^{(1)}(n_3 k_o a) - H_1^{(1)'}(n_3 k_o a) Y_1(n_2 k_o a)}{J_1(n_2 k_o a) H_1^{(1)'}(n_3 k_o a) - J_1'(n_2 k_o a) H_1^{(1)}(n_3 k_o a)} \quad (3)$$

In general, the wave number k_i is a complex number given by $k_i = 2\pi n_i / \lambda$ with $i=1, 2, 3$. The eigenvalue equation for TE modes requires the use of numerical techniques for the determination of complex roots in order to determine the different wavelengths of resonances. We can consider that the optical resonances inside the microcapillary must satisfy a resonance condition $m\lambda_{\text{ress}} = n_{\text{eff}}(2\pi r)$, where m is the modal order number, n_{eff} is the effective refractive index of the mode and r represents the average radius of the circle formed by the optical path length of the ray through the cross-section of the capillary. In the case of TM modes, the magnetic field is along the axial axis of the cylinder, in this case in the differential equation that solves the problem it is assumed that ε it does not present azimuthal dependence

$$\nabla^2 H_z(r, \phi) - \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial r} \frac{\partial H_z(r, \phi)}{\partial r} + k_o^2 n^2 H_z(r, \phi) = 0 \quad (4)$$

The solution of this equation is again a set of Bessel functions for the magnetic field in each of the regions, which presents the form

$$H_z(mk_o r) = \begin{cases} a_1 J_1(n_1 k_o r) & r \leq b \\ b_1 J_1(n_2 k_o r) + c_1 Y_m(n_2 k_o r) & b < r \leq a \\ d_1 H_1^{(1)}(n_3 k_o r) & r > a \end{cases} \quad (5)$$

$$\frac{n_2^2 J_1'(n_1 k_o b) Y_1(n_2 k_o b) - n_1^2 J_1(n_1 k_o b) Y_1'(n_2 k_o b)}{n_1^2 J_1(n_1 k_o b) J_1'(n_2 k_o b) - n_2^2 J_1'(n_1 k_o b) J_1(n_2 k_o b)} = \frac{n_3^2 H_1^{(1)}(n_3 k_o a) Y_1'(n_2 k_o a) - n_2^2 Y_1(n_2 k_o a) H_1^{(1)'}(n_3 k_o a)}{n_2^2 J_1(n_2 k_o a) H_1^{(1)'}(n_3 k_o a) - n_3^2 H_1^{(1)}(n_3 k_o a) J_1'(n_2 k_o a)} \quad (6)$$

In figure 1 a diagram of a capillary is observed which is pressurized in its interior with a fluid at a constant pressure p . This pressurization creates radial forces that are uniformly distributed to the interior of the microcapillary, so that when the capillary is pressurized, a uniform positive circumferential displacement will occur at the inner and outer radius. It is possible to determine the circumferential displacement of the radius in terms of the characteristics of the material by knowing the elasto-optical phenomenon experienced by the capillary. In figure 1, $u(b, p)$ is the circumferential displacement of the internal radius shown in the discontinuous curve and $u(a, p)$ is defined as the circumferential displacement of the external radius, also observed in the external discontinuous curve, g represents the thickness of the capillary. In order to solve the equation of eigenvalues equation 3 and equation 6 of TE and TM modes which are excited to the interior of the cavity, it is necessary to determine the circumferential displacements of the internal and external radius in the microcapillary as a consequence of the pressurization. In this way, the internal and external radius of the microcapillary is given by the expression $b(p) = b_o + u(b, p)$ and $a(p) = a_o + u(a, p)$ respectively, where b_o and a_o

are the internal and external radii without pressurizing. During the computational simulation to determine the roots of the eigenvalue equations of the WGMs modes, it is necessary to set some capillary parameters, such as the material that composes the microcavity and for this we have chosen the material PMMA (Polymethyl-methacrylate) of which some experimental results have been reported [17]. For the PMMA, the circumferential displacement when the capillary is pressurized can be determined from the theory of elasticity assuming a linear behavior according to Hooke's law.

$$u(b, p) = u(r, p)|_{r=b} = \frac{pb}{E} \left(\frac{a^2 + b^2}{a^2 - b^2} + \nu \right) \quad (7)$$

$$u(a, p) = u(r, p)|_{r=a} = \frac{2pb^2a}{E(a^2 - b^2)} \quad (9)$$

Where, r is the radial distance measured from the axial, E is the Young module and ν is the Poisson coefficient. In the same way, the theoretical sensitivity can be determined for different azimuth and radial order numbers for the different TE and TM modes. In the analysis of the results, was possible to numerically determine the sensitivity for microcapillaries with different geometric parameters and were compared with some experimental results obtained by some authors [17]. In Table 1, the theoretical sensitivity determined numerically for three different microcapillaries with different geometric parameters for TE and TM modes is observed and different azimuth order numbers. In the results of the table, the geometrical parameters of the PMMA capillary optical micro-resonators that were simulated, have been chosen according to the geometric characteristics of some capillary tubes that have been manufactured by some authors and whose results have been reported in the literature [17]. the numerical simulation for the calculation of sensitivity, the following parameters were taken into account: $n_1 = 1.000299$, $n_2 = 1.49$, $n_3 = 1.0002924$, $m = 2000$, $\nu = 0.37$ and $E = 2.2 \times 10^8$ Pa

Table 1. Theoretical sensitivity determined numerically for three different microcapillaries with different geometric parameters for TE and TM modes.

CAPILLARIES	EXPERIMENTAL SENSITIVITY (nm/Bar). [17]	INTERNAL RADIUS (μm)	THICKNESS (μm)	THEORETICAL SENSITIVITY (nm/Bar).	
				EM MODE	TM MODE
No 1	0.147	296.70	58.88	0.165 (l=1)	0.159 (l=1)
				0.154 (l=3)	0.163 (l=3)
				0.203 (l=10)	0.158 (l=10)
				0.156 (l=50)	0.158 (l=50)
No 2	0.024	71.30	82.82	0.020 (l=1)	0.019 (l=1)
				0.019 (l=3)	0.019 (l=3)
				0.019 (l=10)	0.019 (l=10)
				0.018 (l=50)	0.019 (l=50)
No 3	0.351	832.96	79.93	0.3466 (l=1)	0.3681 (l=1)
				0.3659 (l=3)	0.3207 (l=3)
				0.3584 (l=10)	0.3601 (l=10)
				0.3417 (l=50)	0.3427 (l=50)

In figure 2, shows the numerical and experimental results of the sensitivity for capillary No 1. During the simulation, it was possible to study capillaries with different geometric parameters

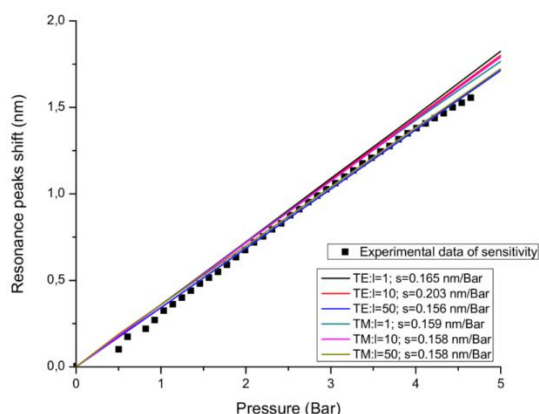


Figure 2. Numerically obtained sensitivity of capillary No. 1 with a wall thickness of $58.88 \times 10^{-6} m$ and internal radius of $296.70 \times 10^{-6} m$ for different modal orders. TE and TM Modes

These results show that TE modes in this type of cavities may experience better sensitivity levels than TM modes, therefore it is necessary to use experimental techniques that allow to control the states of polarization for the modes TM. In the same way, although the theoretical model does not allow to determine the spectrum obtained at the exit of the capillary resonator, it is possible to verify that the sensitivity obtained experimentally presents a good approximation with the numerical results for the modes TE and TM. On the other hand, during the theoretical analysis, it was determined that the Q factor and the losses associated to the dispersion phenomena and the light attenuation in the cavity are not important factors in the measurement applications because the sensitivity of the device is determined from the spectral shifts of resonance wavelengths. During the theoretical development, it was possible to develop a computational simulation using the finite element method which allows observing the coupling of the field to the interior of the cylindrical optical cavity using the program COMSOL Multiphysics 5.1, as seen in the 3D image of figure 3.

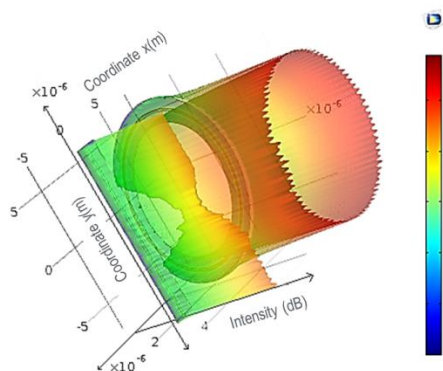


Figure 3. Optical field coupled to the interior to the cavity cylindrical of $16 \times 10^{-6} m$ of external diameter through a cylindrical waveguide of $2 \times 10^{-6} m$ of external diameter. Light enters from the bottom in the graph.

During the simulation, a cylindrical waveguide with an external diameter of the order of $2 \times 10^{-6} m$ couples the field into a cylindrical microcavity with an outer diameter of $16 \times 10^{-6} m$, the refractive index of the core of the cylindrical waveguide is 1.52, while the refractive index of the cladding is 1.5.

4. Conclusions

The measurement of the sensitivity of an optical sensor is an important parameter because it determines the ability of the device to respond to the variation of some external parameter, in this sense, of the results obtained in the development of the present investigation It is shown that micro-resonators manufactured in the form of capillary tubes from certain polymers, can experience a good sensitivity for the measurement of hydrostatic pressure changes inside the resonant cavity and can improve the sensitivity reported by some authors in systems where it is required to measure the hydrostatic pressure of small amounts of a fluid. The sensitivity of the device depends on the

geometrical parameters of the structure, such as the external, internal diameter and the wall thickness of the cavity.

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