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# Reconstruction of a heat transfer coefficients by using FWA approach

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**Abstract**—A parallelized numerical method based on Fireworks Algorithm (FWA) has been developed to solve the Inverse Heat Transfer Problem. The Heat Transfer Coefficient functions obtained on the surfaces of an axisymmetric workpiece as a function of time are estimated by applying the novel technique. The objective function to be minimized by the FWA approach is defined by the deviation of a virtually acquired and the calculated temperatures. Numerical results have been demonstrated in 3 case studies.

**Keywords**—Fireworks Algorithm, IHCP, Heat Transfer Coefficients

## I. INTRODUCTION

The knowledge of the thermal boundary conditions helps to understand the heat transfer phenomena took place during heat treatment processes. Heat Transfer Coefficients (HTC) describes the heat exchange between the surface of an object and the surrounding medium.

The direct problem of one-dimensional heat conduction in a rod with time-varying HTC only approximated or numerical solutions could be found in the literature [18, 19, 20, 21] before 2010. Using a dimensionless temperature and it subtracting an ambient temperature over the surrounding temperature, Lee et al. [22] in 2010 proposed a possible solution form for this system by a shifting function procedure which does not require any integral transform. After some time Lee and Tu [23] have developed another analytical solution for the same system, but they applied a non-homogenous time-varying boundary condition at one end and homogeneous boundary condition with time-varying HTC at the other end instead of the homogeneous rod both ends.

A short time ago, Chiba [26] enlarged the same method to solve the one-dimensional n-layer slab with time-varying HTC in the outer surface. Concerning the indirect solution of one-dimensional heat conduction problems for a slab with time-varying HTC at one end along with an insulated boundary at the other end, just some literature [27 - 31] could be found. In 2000, Chantasiriwan [27] combined the sequential with piecewise function method and the linear boundary conditions to calculate the boundary heat flux and temperature. It used to estimate the time-varying HTC inversely.

The determination of HTC [1] faces a typical Inverse Heat Conduction Problem (IHCP). The IHCP methods are using the thermal samples acquired and predicted by simulations at a location of the work-piece. Several IHCP approaches are based on optimization methods, where the objective function that has to be minimized is determined as the difference of the recorded and calculated temperature data [2, 3]. Genetic algorithms [4, 5, 6] are applied successfully for solving many types of IHCP cases.

The Fireworks Algorithm (FWA) became popular in recent years due to its ability to maintain a right balance between convergence and diversity [7, 8]. The efficiency of FWA technique in inverse heat conduction analysis was not analyzed yet before. We have shown that FWA can reduce the stability problems of the classical methods, for solving the inverse heat conduction problems.

In this work, an IHCP analysis of a time-dependent one-dimensional HTC(t) is presented. Cooling curves measured at the centerline of a cylinder have been used to obtain the inverse heat transfer computation. The objective function which is defined by the quadratic residual between the measurements and the calculated temperatures is minimized. The optimization techniques have been parallelized and implemented on a CPU architecture. Numerical results are demonstrated that HTC functions can be performed by using the proposed approach.

## II. THE INVERSE HEAT TRANSFER PROBLEM

Assuming that the temperature on its surface is theoretical set during the heat transfer process, it is possible to solve the inverse heat conduction problem by determining the temperature variations of the thermal boundary conditions [1, 2, 3]. The temperature is given at  $p$  points in the solid region. Calling  $T_k^m$ , the theoretical set temperatures, and  $T_k^c$ , the calculated temperatures at those points, the solution of the present inverse problem can be obtained by minimizing the following fitness function

$$S = \sum_{k=1}^p (T_k^m - T_k^c)^2 = \min \quad (1)$$

The inverse problem is recast as an optimization problem. A variety of numerical and analytical techniques have been developed to solve the optimization problems.

## III. THERMAL FIELD CALCULATION

A one-dimensional axisymmetric heat conduction model is considered to estimate the temperature distribution in a cylindrical work-piece. Both the thermal conductivity, density, and the heat capacity are assumed as functions of the temperature, respectively  $k(T)$ ,  $\rho(T)$  and  $C_p(T)$ . The one-dimensional mathematical formulation of this nonlinear transient heat conduction problem can be described as follows:

$$\frac{\partial}{\partial r} \left( k \frac{\partial T}{\partial r} \right) + \frac{k}{r} \frac{\partial T}{\partial r} + q_v = \rho \cdot C_p \frac{\partial T}{\partial t} \quad (2)$$

while initial and boundary conditions are

$$T(r, t = 0) = T_0 \quad (3)$$

$$k \frac{\partial T}{\partial r} = HTC(t) [T_q - T(t)] \quad (4)$$

where  $r$  is the radius,  $t$  is time,  $T_0$  is the initial temperature,  $T_q$  is the temperature of the cooling medium

and  $q_v$  stands for volumetric heat generation. It has to be noted that the phase transformations of the materials applied do not occur during the experiments. Therefore latent heat generation induced by phase transformations is not considered ( $q_v=0$ ).

#### IV. FIREWORKS ALGORITHM

The basic FWA model [7] consists of fireworks and number of sparks (s) which are placed around firework. They are moving in the search space. For a N-dimensional search space, the position of the  $i^{\text{th}}$  spark is represented as  $X_i = (x_{i1}, x_{i2}, \dots, x_{iN})$ . Each spark is a potential solution of the global optimum. In other words, each spark stands for a set of input parameters by its position ( $X_i$ ) which could give the lowest fitness value (the global optimum) in the search space. There are more than one firework population is calculating in the space at the same time. At each generation, the new spark position is found by adding a displacement to the current position and leave intact the best fitness value of spark to memorize the best position. There are two types of sparks in the algorithm [9, 10, 11]. The first is the “explosion sparks”, the second is the “gaussian sparks”.

The explosion sparks displacement each dimension is calculated by

$$d_i = A_i \cdot r \quad (5)$$

where  $r$  is a uniform distributed random number between -1 and 1. The  $A_i$  is the amplitude. The next position of spark is

$$X_{i+1} = X_i + d_i \quad (6)$$

The amplitude is calculated by

$$G = \sum_{N=1}^{i-1} (f(X_i) - y_{\min}) \quad (7a)$$

$$A_i = \begin{cases} \hat{A} \cdot \frac{f(X_i) - y_{\min}}{G}, & \text{if } G \neq 0 \\ 0, & \text{if } G = 0 \end{cases} \quad (7b)$$

where the  $\hat{A}$  constant to control the explosion amplitude. The  $y_{\min} = \min(f(X_i))$  means the minimum value of fitness function. The  $f(X_i)$  the actual spark value of fitness function. Each spark has a minimum amplitude value which depends on the actual iteration and the highest iteration number (6)

$$A_{\min} = A_{\text{init}} - \frac{A_{\text{init}} - A_{\text{final}}}{t_{\text{max}}} \cdot \sqrt{((2 \cdot N_{\text{max}} - n) \cdot n)} \quad (8)$$

where  $N_{\text{max}}$  is the maximum iteration number and  $n$  is the actual iteration number. The “gaussian spark” displacement is calculated by

$$d_i = g \cdot (X_k - X_i) \quad (9)$$

where the  $g$  is a Gaussian distributed random number. The  $X_k$  is also randomly selected firework position in the space, and the  $X_i$  is the spark position. After iteration in every population, the best fitness value ( $F_{\text{best}}$ ) and position ( $X_{\text{best}}$ ) of spark are selected.

#### V. COMPUTATION PROCEDURE FOR THE FIREWORKS ALGORITHM

The inverse analysis aims to iteratively estimate the unknown HTC using the FWA procedure which results in an acceptable difference between measurements taken at the given locations of the workpiece and temperatures estimated from the numerical model. The fitness function value of each spark at the  $n^{\text{th}}$  iteration is given by the difference between the theoretical set and calculated temperature curves, (4) at the position  $X_i^n$ . The

computational steps of the FWA algorithm described above are given as follows:

- Step 1: Generate the initial sparks by random position, each spark are moving a dedicated population
- Step 2: Evaluate the fitness function of each spark.
- Step 3: Update the  $F_{\text{best}}$  and  $X_{\text{best}}$  for each population, if its fitness is smaller than the fitness of its previous best position.
- Step 4: Update the firework position as the best spark position.
- Step 5: Update each spark according to (4 - 9).
- Step 6: Repeat the loop until the stopping criteria or a predefined number of generations is reached ( $N_{\text{max}}$ ).

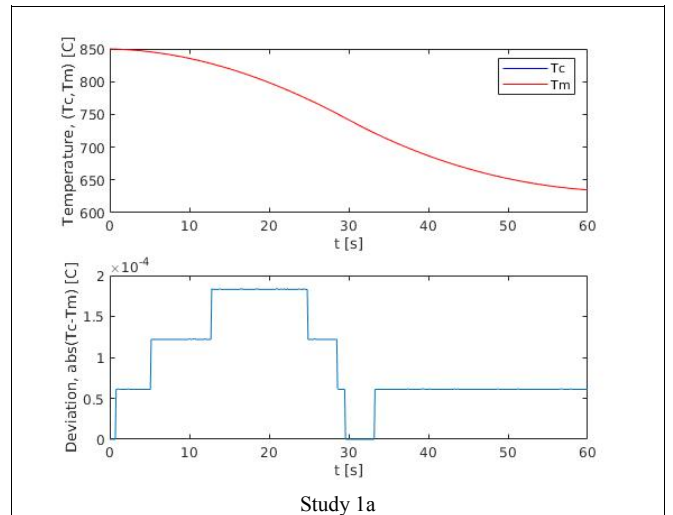
It is strongly advised to parallelize the computational jobs in Step 2 because there are no need interferences between the iterations in each cycle as well as there is no communication between the sparks. The parallelizing the algorithm speeds up the computations by the proportion of the number of threads. The FWA algorithm process diagram can be shown on Fig. 3.

#### VI. CASE STUDIES

The HTC obtained during immersion quenching of a cylindrical bar have been estimated by using the theoretical approach. The diameter of the cylindrical workpiece is assumed to 6.25 mm. The workpiece has been heated up to 880°C and immersed in cooling material without agitation at 20°C temperature. The cooling time is 60 sec.

A 1D axisymmetric heat transfer model was applied to calculate the temperature distribution during the cooling process ( $T_k^m$ ). Inverse computations have been carried out by including FWA algorithm, to predict the HTC(t) functions. The  $T_k^c$  has been calculated by this predicted HTC(t) function. The calculation process in each FWA stopped when the relative deviation of the fitness value was less than 0.1.

The theoretical HTC functions are defined as HTC functions which are used to calculate the  $T_k^m$  values in formula (1). The theoretical HTC will help to prove the FWA algorithm is working. The theoretical HTC function in the first study has 3 values: (500, 5000, 800) W/(m<sup>2</sup>K), the second study has 5 values: (200, 500, 4000, 2000, 800) W/(m<sup>2</sup>K) and the third study has 10 values: (100, 300, 800, 1000, 6000, 5000, 4000, 2000, 1200, 800) W/(m<sup>2</sup>K).



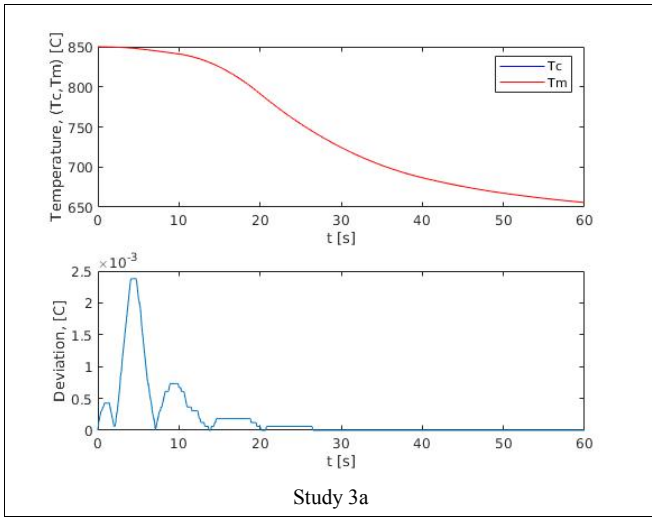
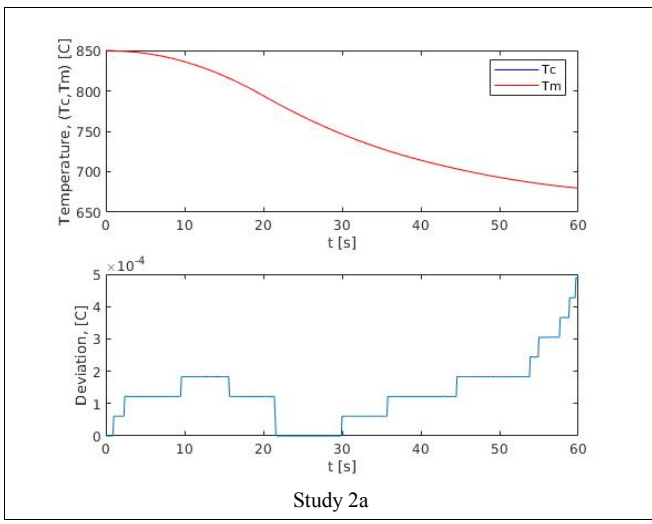


Fig. 1. The cooling curves of the theoretical set  $T_k^m$ , and the predicted  $T_k^c$  samples and their deviations (Study 1a, Study 2a, Study 3a)

The theoretical set  $T_k^m$ , and the reconstructed  $T_k^c$  cooling curves are shown in Fig. 1. Satisfactory agreement of the original and the predicted cooling curves can be observed. The difference between the theoretical set and estimated samples as a function of time is shown in the charts of Fig. 1. In the study 1b the HTC function has only three values. In the study 2b the HTC function has five values, and in the study 3b the HTC function has ten values. The theoretical and the reconstructed HTCs and their deviations are shown in Fig. 2.

To quantify the magnitude of deviation between the theoretical set and the recovered temperature samples the mean, standard deviation and maximum value of the difference of cooling curves in each study were calculated (Table 1).

Using the FWA algorithm for estimating the HTC function in a one-dimensional axisymmetric model could be precise in the examined cases between calculated and theoretical cooling curves. On the other hand, the deviation of HTC functions not as same as accurate as the deviation of cooling curves. More investigations need to determine a modified procedure of FWA algorithm to more complex HTC functions could be calculated less or equal of calculated deviations such as one- or two-dimensional axisymmetric cases (Fig. 1, 2). The convergence of the applied algorithm has been experienced all times but further developments need to guarantee that. The speed of convergence of this FWA algorithm is strongly depending on the parameters of algorithm.

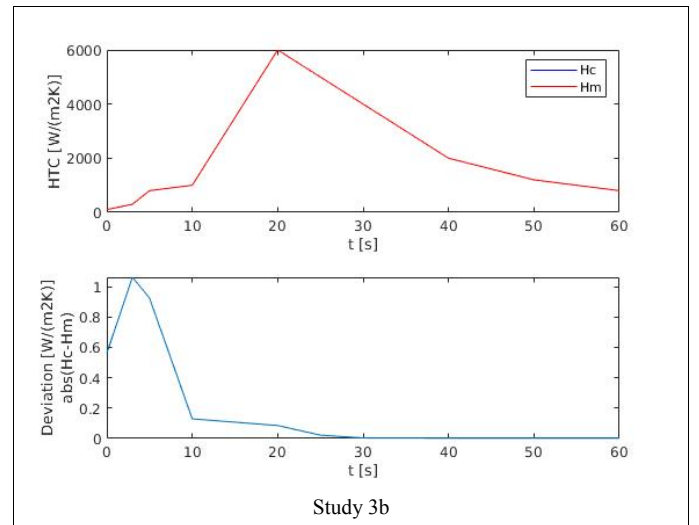
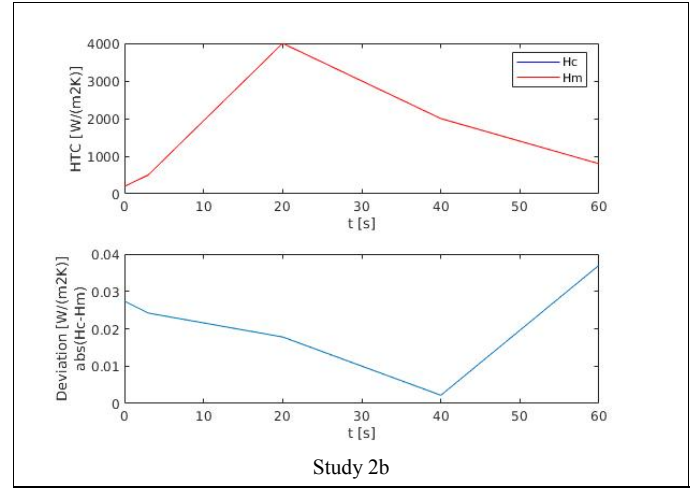
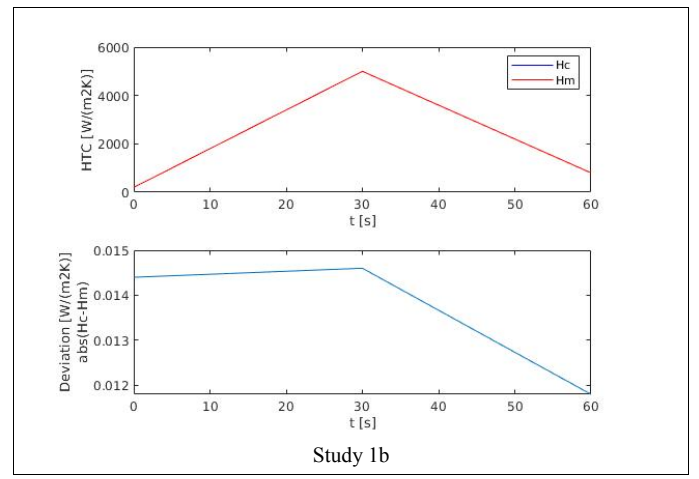


Fig. 2. Theoretical and predicted HTC functions and their deviations (Study 1b, Study 2b, Study 3b)

TABLE I. THE STATISTICAL INFORMATION OF DEVIATIONS BETWEEN THEORETICAL SET  $T_k^m$ , AND THE RECONSTRUCTED  $T_k^c$  COOLING CURVES

Study	Mean deviation [°C]	Standard deviation, [°C]	Maximum deviation, [°C]
1	3.8e-05	0.0001	0.0002
2	4.0e-05	0.0001	0.0004
3	6.3e-05	0.0004	0.0024

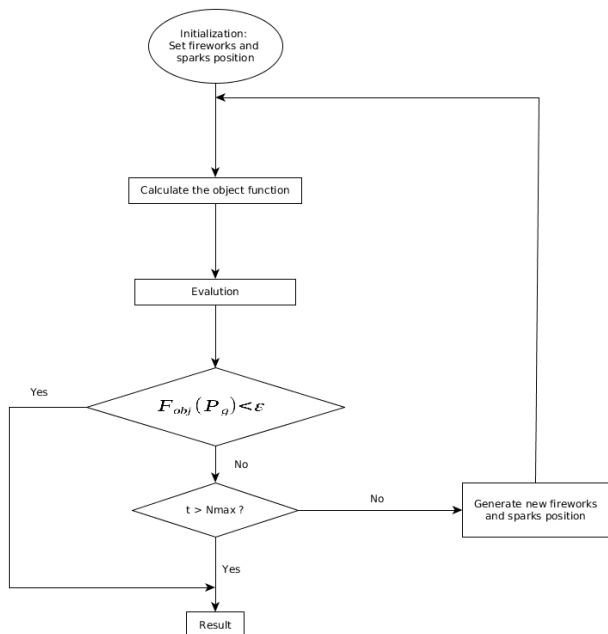
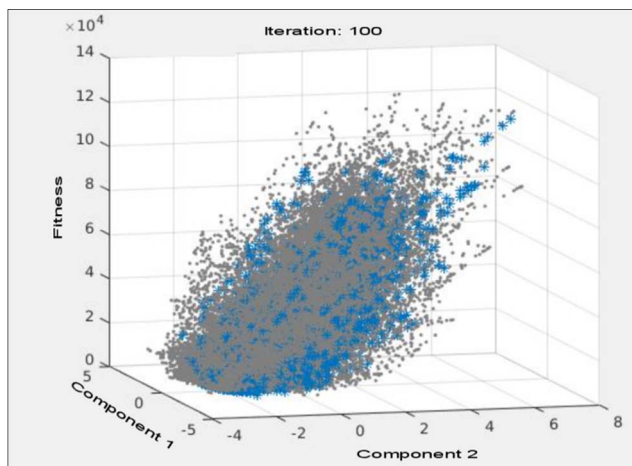
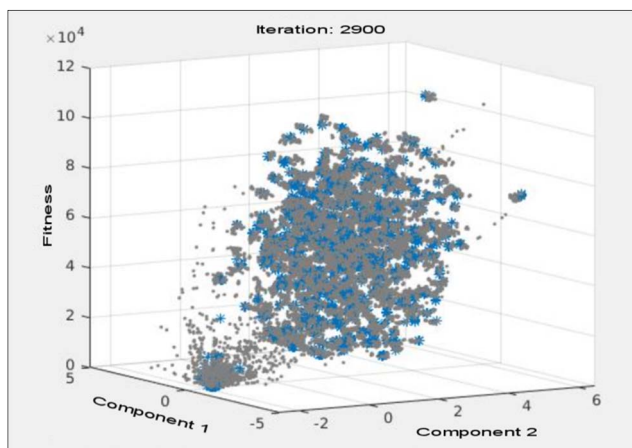


Fig. 3. The FWA algorithm process diagram

To show, how does FWA algorithm work, 3 shots were created. First figure shows the FWA is started and second almost the end of the calculation. The shots only show the last 50 iteration. The blue stars represent the fireworks position; the red stars represent the best sparks position. The FWA is calculating the HTC function with 10 values. The figures are represented a 10 dimension space which was converted to 2 dimension one with principal component analysis. The “component 1” and “component 2” represent the first 2 components from the PCA. The third dimension on the figures is set to the fireworks and sparks fitness value. (Fig. 4a-c)



a



b

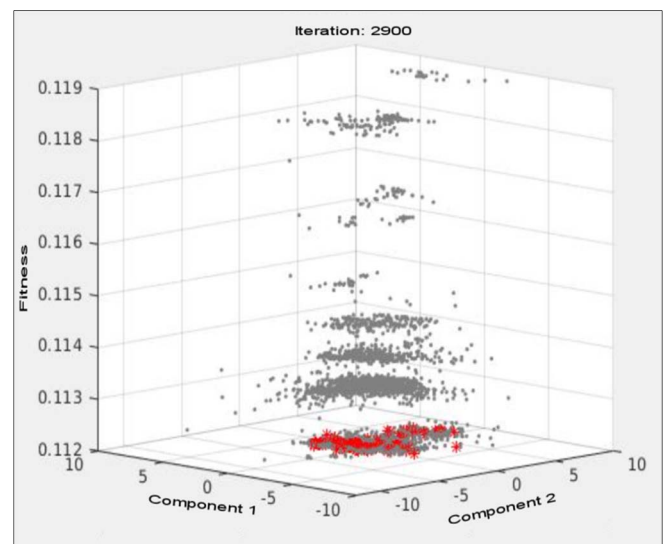


Fig. 4. Three states of the FWA algorithm: a) at iteration 100, b) at iteration 2900, c) at iteration 2900 but only the last 50 best sparks show

## VII. CONCLUSIONS

An inverse thermal analysis using Fireworks Algorithm has been presented to estimate the Heat Transfer Coefficient in a one-dimensional heat conduction problem. The HTC obtained on the surfaces of a cylindrical work-piece was considered as functions of local coordinates and time/temperature. The temperature signals acquired during immersion quenching experiment have been applied for the inverse method. The obtained results underline the feasibility of the procedure and the capabilities for the FWA technique to predict a complex Heat Transfer Coefficients without using any prior information of the unknown transient functions.

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