Fuzzy Truth Maintanance System for Nonmonotonic Reasoning

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November 16, 2022

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***Abstract*— Monotonic logic has conclusion. non-monotonic has different conclusions. The non-monotonic problem is undecided. Fuzzy logic will give conclusion for non-monotonic problems. In this paper, . Fuzzy logic is used to quantify undecided problem into decidable problem. Fuzzy non-monotonic reasoning is studied with two membership functions unknown and known twofold fuzzy set to made undecided problem in to decidable. Fuzzy truth maintenance system (FTMS) is studied for computation of fuzzy non-monotonic reasoning to made monotonic. Some examples are given for fuzzy non-monotonic reasoning .**

Keywords—non-monotonic reasoning, fuzzy Sets, twofold fuzzy sets, fuzzy non-monotonic reasoning, FTMS, incomplete knowledge

I. INTRODUCTION

Sometimes Artificial Intelligence (AI) has to deal with undecided problems. Non-monotonic problem is undecided. In non.-monotonic reasoning, if some knowledge is added to the system then the conclusion will be changed, if knowledge base is incomplete then the inference is also incomplete. Knowledge base is collecting the facts and gives the conclusion based on facts. In monotonic reasoning, conclusion can't be changed if knowledge is added. In non-monotonic reasoning, if some knowledge is added to the system then the conclusion is changes. In non-monotonic logic, if some knowledge is added to system than inference will be changed such reasoning falls under undecided problems. In undecided problems, unknown information is more then known information.

John McCarthy [5] formalized non-monotonic reasoning with predicates P(x1,x2,…,xn) for the propositions of type "x is A".

The non-monotonic logic may be defined as

$\forall x\ (P(x) \wedge Q(x) \rightarrow R(x))$

$\exists P(x) \wedge Q(x) \rightarrow \neg R(x)$

For instance ,

$\forall x$ (bird(x) Λ wings(x) → fly(x))

bird(peacock) Λ wings(peacock) → fly(peacock)

bird(penguin) Λ wings(penguin → ┐ fly(penguin

$\exists x$ (bird(x) Λ wings(x) Λ unknown(x)) → ┐ fly(x))

$\exists x$ (bird(x) Λ wings(x) Λ known(x)) → fly(x))

There is conflict with un{known, known}.

The two fold fuzzy set may be defined with {unknown, known}.

I. FUZZY LOGIC

formation like Probability, Dempster- Shaffer theory, Possibility, Plausibility etc. Zadeh [14] fuzzy logic is based on unknown rather than probable (likelihood). The fuzzy logic made imprecise information in to precise.

Zadeh[17] fuzzy logic is defined with single membership function.

The set A of X is characterized by its membership function $\mu_A(x)$ and ranging values in the unit interval [0, 1]

$\mu_A(x): X \rightarrow [0, 1],\ x \in X$, where X is Universe of discourse.

$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(a_n)/x_n$, where "+" is union

$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)=$ 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos

$\mu_{\neg A}(x)= 1-\mu_A(x)$ Negation

$\mu_A(x) \wedge \mu_A(x) =\min\{ \mu_A(x) \wedge,\mu_A(x)\}$ Conjunction

$\mu_A(x) \vee \mu_A(x) =\max\{\mu_A(x),\mu_A(x) \}$ Disjunction

$\mu_{QA}(x) =\mu_A(x)\ q$ Quantifier

where Q is very, more or less and q is real value

The fuzzy rules are of the form "if <Precedent Part> then <Consequent Part>"

if x is P the n x is Q.

. if x is P_1 and x is P_2 …. x is P_n then x is Q

The Zadeh [14] fuzzy conditional inference s given by

if x is P_1 and x is P_2 …. x is P_n then x is Q =

$$\min 1, (1-\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x)) + \mu_Q(x)\} \quad (2.1)$$

The Mamdani [8] fuzzy conditional inference s given by

if x is P_1 and x is P_2 …. x is P_n then x is Q =

$$\min \{\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x) , \mu_Q(x)\} \quad (2.2)$$

The fuzzy conditional inference may be derived "Consequent Part" from "Precedent Part" . [10].

if x is P_1 and x is P_2 …. x is P_n then x is Q =

x is P_1 and x is P_2 …. x is P_n

Fuzzy conditional inference is given by fuzzy conditional inference

$$\mu_Q(x), = \min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x)) \quad (2.3)$$

.

$P(x) \wedge Q(x) \rightarrow R(x)$

$R(x) =P(x) \wedge Q(x)$

For instance,

x is bird Λ x has wings → x can fly

x can fly= x is bird Λ x has wings

II. FUZZY NON-MONOTONIC LOGIC

Zadeh [16] Fuzzy logic will made undecided into decidable. Fuzzy non-monotonic reasoning. May be formalized with fuzzy predicate for the proposition of type " x is A", where A is fuzzy set may be defined as

$\forall x\ (P(x) \rightarrow R(x))$

$\forall x\ (P(x) \rightarrow \neg Q(x)$

For instance ,

$\forall x$ (bird(x) → fly(x))

$\forall x$ (bird(x) → ┐ fly(x)

$\forall x$ (tall(x) → run-fast(x))

$\forall x$ (tall(x) → ┐ run-fast(x)

x is bird Λ x is unknown to fly → x can fly

Suppose ,

x is bird Λ x is known to fly → x can 't fly

or

x is bird Λ x is unknown to fly → x can't fly

For example,

Ozzie is bird Λ Ozzie x is unknown to fly → Ozzie can fly

Ozzie is bird Λ Ozzie x is known to fly → Ozzie can't fly

Ozzie is bird Λ Ozzie is unknown to fly → Ozzie can fly

The fuzzy non-monotonic logic may be defined as

$\forall x\ (P(x) \wedge Q(x) \wedge Q1(x) \rightarrow R(x))$

$\forall x\ (P(x) \wedge Q(x) \wedge Q2(x) \rightarrow \neg R(x)$

For instance ,

$\forall x$ (bird(x) Λ wings(x) Λ unknown-of-fly(x) → fly(x))

$\forall x$ (bird(x) Λ wings(x) Λ known-of-fly(x) → ┐ fly(x)

III. FUZZY NON-MONOTONIC LOGIC

Zadeh [17] Proposed fuzzy set with single membership function. The two fold fuzzy set [12] will give more evidence than single membership function.

The fuzzy non-monotonic set may defined with two fold membership function using unknown and known

Definition: Given some Universe of discourse X, the proposition " x is P" is defined as its two fold fuzzy membership function as

$\mu_P(x) = \{\mu_P^{unknown}\ (x),\ \mu^{known}\ (x)\}$

Interpreting "truth is knkown but KNOWN is known", the twofold fuzzy set is given by

$P = \{\mu_P^{unknown}(x),\ \mu_P^{known}(x)\}$

Where P is Generalized fuzzy set and x Є X ,

$0 <= \mu_P^{unknown}(x) <=1$ and, $0 <= \mu_P^{known}(x) <=1$

$P = \{\ \mu_P^{unknown}(x_1)/x_1 + \ldots + \mu_P^{unknown}(x_n)/x_n,$
$\mu_P^{known}(x_1)/x_1 + \ldots + \mu_P^{unknown}(x_n)/x_n$, x_i Є X, "+" is union

For example ' x will fly" , fly may be given as

Bird = $\{\mu_{bird}^{unknown}(x),\ \mu_{bird}^{known}(x)\}$

Suppose P and Q is fuzzy non-monotonic sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Negation

$P' = \{1-\ \mu_P^{unknown}(x),\ 1-\ \mu_P^{known}(x)\quad \}/x$

Disjunction

$PVQ=\{\ \max(\mu_P^{known}\ (x)\ ,\ \mu_P^{known}\ (y)),\ \max(\mu_Q^{unknown}\ (x)\ ,\ \mu_Q^{unknown}\ (x))\}$

Conjunction

$P\Lambda Q=\{\ \min(\mu_P^{known}\ (x)\ ,\mu_P^{known}\ (y)),\ \min(\mu_Q^{unknown}\ (x)\ ,\ \mu_Q^{unknown}\ (x))\ \}$

Implication

Zadeh fuzzy conditional inference

$P\rightarrow Q=\{\min(1,\ 1-\ \mu_P^{known}(x) +\ \mu_Q^{known}(x)\ ,\ \min(\ 1,\ 1-\ \mu_P^{known}\ (x) + \mu_Q^{known}(x)\}$

Mamdani fuzzy conditional inference

$P\rightarrow Q=\{\min(\ \mu_P^{unknown}(x)\ ,\ \mu_Q^{unknown}(y)\ ,\ \min(\ \mu_P^{known}(x)\ ,\ \mu_Q^{known}\ (x)\}$

Reddy fuzzy conditional inference

$P\rightarrow Q=\{\min(\ \mu_P^{unknown}(x)\ ,\ \mu_P^{known}(y)\}$

Composition

$P \circ R = \{\min_x(\ \mu_P^{unknown}(x),\ \mu_P^{unknown}\ (x)\ ),\ \min_x(\ \mu_R^{known}(x),\ \mu_R^{known}\ (x)\ )\}$

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

Concentration

"x is very P

$\mu_{very\ P}(x) = \{\ \mu_{very\ P}^{unknown}\ (x)^2,\ \mu_{very\ P}^{known}\ (x)\mu_P(x)^2\ \}$

Diffusion

"x is more or less P"

$\mu_{more\ or\ less\ P}(x) = (\ \mu_{more\ or\ less}^{unknown}\ (x)^{1/2},\ \mu_{more\ or\ less}^{known}\ (x)\mu_P(x)^{0.5}$

$\mu_P(x)^{(unknown,\ known)} \Lambda\ \mu_Q(x)^{(unknown\text{-},\ known)} \rightarrow \mu_S(x)^{(unknown,\ known)}$

where P,Q and S are twofold fuzzy set known, known}.

$\mu_{bird}(x)\ \Lambda\ \mu_{wings}(x) \rightarrow \mu_{fly}(x)$

$\mu_{bird}(x)^{(unknown,known)} \Lambda \mu_{wings}(x)^{(unknown\text{-},known)} \rightarrow \mu_{fly}(x)^{(unknown,\ known)}$

The conflict of the incomplete information may be defend by fuzzy certainty factor(FCF).

FCF =(unknown- known)

$\mu^{FCF}{}_P(x) \rightarrow [0,1],\ x \in X$

$\mu_{bird}{}^{FCF}\ (x) \rightarrow [0,1]$

Where known and unknown are the fuzzy membership functions.

The fuzzy non-monotonic reasoning will bring uncertain knowledge in to certain knowledge. In undecided problems, unknown is more than the known information.

$\mu_{bird}(x)^{\ (unknown,\ known)} \rightarrow \mu_{fly}(x)$

$\mu_P(x)^{\ (unknown,\ known)} \rightarrow \mu_S(x)$

S= $\mu_S{}^{FCF}\ (x\ ) = 1 \quad \mu_P{}^{FCF}\ (x >= \alpha,$

$0 \quad \mu_P^{FCF}(x) < \alpha$

fly= $\mu_{fly}^{FCF}(x) = 1 \quad \mu_{bird}^{FCF}(x >= 0.6$
$0 \quad \mu_{bird}^{FCF}(x) < 0.6$

Bird = {0.2/penguin +0.3/Ozzie+ 0.8/parrot+ 0.9/waterfowl + 1.0/eagle , 0.1/penguin +0.2/Ozzie+ . 0.1/parrot+ 0.1/waterfowl + 0.1/eagle }

Wngs= {0.1/penguin +0.3/Ozzie+ 0.7/parrot+ 0.8/waterfowl + 0.9/eagle , 0.0/penguin +0.1/Ozzie+ . 0.2/parrot+ 0.1/waterfowl + 0.1/eagle }

Definition: fuzzy non-monotonic logic is simply two fold fuzzy logic for the proposition of the type "x is P"

$\mu_P(x) = \{\mu_P^{unknown}(x), \mu_P^{known}(x)\}$

$\mu_{bird}^{FCF}(x) = \{\mu_{bird}^{unknown}(x), - \mu_{bird}^{known}(x)\}$
= {0.2/penguin +0.3/Ozzie+ 0.8/parrot+ 0.9/waterfowl + 1.0/eagle , 0.1/penguin +0.1/Ozzie+ . 0.1/parrot+ 0.1/waterfowl + 0.1/eagle }
= { 0.1/penguin +0.2/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }
$\mu_{wings}^{FCF}(x) = \{\mu_{wings}^{unknown}(x), - \mu_{wings}^{known}(x)\}$

wings= {0.1/penguin +0.3/Ozzie+ 0.7/parrot+ 0.8/waterfowl + 0.9/eagle , 0.0/penguin +0.1/Ozzie+ . 0.2/parrot+ 0.1/waterfowl + 0.1/eagle }
= { 0.1/penguin +0.2/Ozzie+ . 0.5/parrot+ 0.6/waterfowl + 0.8/eagle }

Using (2.3), fuzzy inference is given by
if x is P_1 and x is P_2 …. x is P_n then x is Q

if x is P_1 and x is P_2 …. x is P_n then x is Q_1
$Q_1 = P_1 \wedge P_2 \wedge \ldots \wedge P_n$
The fuzzy non-monotonic logic may be defined as
$\forall x\, (P(x) \wedge Q(x) \wedge Q1(x) \rightarrow R(x))$
$\forall x\, (P(x) \wedge Q(x) \wedge Q2(x) \rightarrow \neg R(x)$
For instance ,
∀x (bird(x) Λ wings(x) Λ unknown-of-fly(x) → fly(x))
∀x (bird(x) Λ wings(x) Λ known-of-fly(x) → ┐ fly(x)

The two statements combined with two fold fuzzy logic.

x is bird Λ x has wings → x can fly
x can fly= x is bird Λ x has wings

$\mu_{bird}(x) \wedge \mu_{wings}(x) \rightarrow \mu_{fly}(x)$
$\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\} \wedge \{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x) = \mu_{fly}(x)$
$\mu_{bird}^{FCF}(x) \wedge \mu_{wings}^{FCF}(x) \rightarrow \mu_{fly}(x)$
if x is P_1 and P_2 …x is P_n then x is P_1 and P_2 …. x is P_n

$= \min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$
$\mu_{fly}(x) = \mu_{bird}^{FCF}(x) \wedge \mu_{wings}^{FCF}(x)$

$\mu_{bird}^{FCF}(x)$ = {0.1/penguin +0.1/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }

$\mu_{wings}^{FCF}(x) \quad \mu_{wings}(x) = \{\mu_{wings}^{unknown}(x), \mu_{wings}^{unnown}(x)\}$
= { 0.1/penguin +0.2/Ozzie+ . 0.5/parrot+ 0.6/waterfowl + 0.8/eagle }

Fuzzy conditional inference "consequent part "may be derived from "precedent part".

$\mu_P(x) \wedge \mu_Q(x) \rightarrow \mu_S(x)$
$\mu_S(x) = \mu_P(x) \wedge \mu_Q(x)$
x is bird Λ x has wings → x can fly
x can fly =min { x is bird , x has wings }
= { 0.1/penguin +0.2/Ozzie+ . 0.5/parrot+ 0.6/waterfowl + 0.8/eagle }

The inference of " x can fly" for $\alpha >= 0.5$ is given by
= 1/parrot+ 1/waterfowl + 1/eagle
Here fuzzy logic made imprecise information to precise information's. Some birds can fly and some birds can't fly.
The fuzzy decision sets or quasi fyzzy set is defined by

R= $\mu_A^R(x) = 1 \quad \mu_A^{FCF}(x) >= \alpha$,
$0 \quad \mu_A^{FCF}(x) < \alpha$

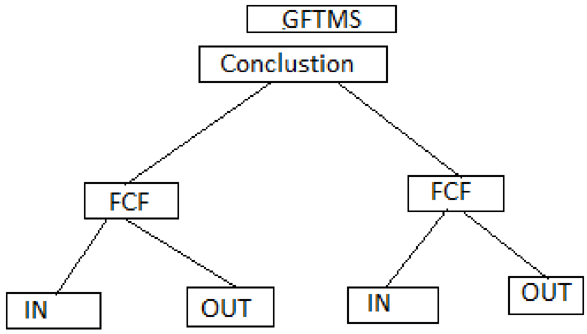
The parrot, waterfowl and eagle can fly.
The penguin and Ozzie can't fly
The inference of " x can.t fly" for $\alpha < 0.5$ is given by
= 0/.1penguin +0.2/Ozzie
The inference of " x can fly" for $\alpha >= 0.5$ is given by
= 0.6/parrot+ 0.6/waterfowl + 0.8/eagle

he parrot, waterfowl and eagle can fly and, penguin and Ozzie are can't fly.

IV. FUZZY TRUTH MAITANACE SYSTEM

Doyel [4] studied truth maintenance system TMS] for non-monotonic reasoning
The fuzzy truth maintenance system (FTMS) for fuzzy non-monotonic reasoning using fuzzy conditional inference as

FTMS is having There is list of justification and conditions.



if x is bird and x has wings then x can fly

List L(IN-node, OUT-node), FCF-node
IN-node evidence is unknown
OUT-node evidence is known
FCF is (unknown-known)
Consider the proposition " if x is bird then x is fly)
x is bird
IN 0.9
OUT 0.1
FCF 0.8
Conclusion : fly (if FCF>=0.5 fly otherwise can't fly)
x is bird
IN 0.3
OUT 0.1
FCF 0.2
Conclusion : can't fly (if FCF>=0.5 fly otherwise can't fly)

Consider the proposition " if x has wings then x is fly)
x has wings
IN 0.8
OUT 0.1
FCF 0.7
Conclusion : fly (if FCF>=0.5 fly otherwise can't fly)
x is bird
IN 0.3
OUT 0.1
FCF 0.2
Conclusion : can't fly (if FCF>=0.5 fly otherwise can't fly)

Consider the proposition " if x is bird and has wings then x is fly)
x is bird
IN 0.9
OUT 0.1
FCF1 0.8
x has wings
IN 0.8
OUT 0.1
FCF 2 0.7
FCF=min{FCF1,FCF2}=0.7
Conclusion : fly (if FCF>=0.5 fly else can't fly)

in the case of parrot, waterfowl and eagle can fly
in case of Penguin and Ozzie can't fly.

Here the fuzzy nonmonotonic logic is making uncertainty to certainty.

V. FUZZY MODULATIONS AND LOGIC PROGRAMMING

The fuzzy reasoning system(FRS) is complex reasoning system for incomplete AI problem solving. The fuzzy predicate logic (FPL) is modulating transform fuzzy facts and rules in to meta form(semantic form). These fuzzy facts and rules are modulated to represent the knowledge available to the incomplete problem [14].

"x is A" is may be represented as
[A]R(x),
where A is twofold fuzzy set {unknown, known}, R is relation and x is individual in the Unversed of discourse X.
For instance
"x is bird" is modulated as
[bird]is(x)

The FPL is e combined with logical operators.
Let A and B be two fold fuzzy sets.
x is ¬A
[¬A]R(x)
x is A or x is B
[A V B]R(x)
x is A and x is B
[A Λ B]R(x)
if x is A then x is B
[A → B]R(x)

x is bird
[bird]is(x)
if x is bird then x can fly
if [bird]is(x) then [fly]is(x)
or
[bird]→[fly]is(x)
if x is bird and x has wings then x can fly
if [bird]is(x)Λ[wings]has[x] then [fly]can(x)

if x is x is P_1 and P_2 …. And x is P_n then Q = { $\mu_Q(x)$}
if [bird]is(x)Λ[wings]has[x] then [fly]can(x)
[fly]can(x)= { [bird]is(x)Λ[wings]has[x]
if [bird]is(x)Λ[wings]has[x] then [fly]can(x)
[fly]cant(x)= { [bird]is(x)Λ[wings]has[x]
The Logic Programming may be written in SWI-Prolog as

```
fuzzy(Ozzie, A,B, F) :- A<B, F is B.

fuzzy(Ozzie, A,B, F) :- A>=B, F is A.

?-run(X,0.7,0.6,F).
F=0.6
If F >=0.5, Ozzie can fly
?-run(Ozzie,,0.5,0.4,F).
F=0.4
If F <0.5, Ozzie can't fly
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VI. MEDICAL APPLICATION

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