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Vasil Penchev

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Quantum phenomenology as a "rigorous science": the triad of epoché and the symmetries of information

Vasil Penchev, vasildinev@gmail.com
Bulgarian Academy of Sciences: Institute of Philosophy and Sociology:
Dept. of Philosophy of Science

Abstract. Huserl (a mathematician by education) remained a few famous and notable philosophical "slogans" along with his innovative doctrine of phenomenology directed to transcend "reality" in a more general essence underlying both "body" and "mind" (after Descartes) and called sometimes "ontology" (terminologically following his notorious assistent Heidegger). Then, Husserl's tradition can be tracked as an idea for philosophy to be reinterpreted in a way to be both generalized and mathenatizable in the final analysis. The paper offers a pattern borrowed from the theory of information and quantum information (therefore relating philosophy to both mathematics and physics) to formalize logically a few key concepts of Husserl's phenomenology such as "epoché" "eidetic, phenomenological, and transcendental reductions" as well as the identification of "phenomenological, transcendental, and psychological reductions" in a way allowing for that identification to be continued to "eidetic reduction" (and thus to mathematics). The approach is tested by an independent and earlier idea of Husserl, "logical arithmetic" (parallelly implemented in mathematics by Whitehead and Russell's *Principia*) as what "Hilbert arithmetic" generalizing Peano arithmetics is interpreted. A basic conclusion states for the unification of philosophy, mathematics, and physics in their foundations and fundamentals to be the Husserl tradition both tracked to its origin (in the being itself after Hidegger or after Husserl's "zu Sache selbst") and embodied in the development of human cognition in the third millennium.

Keywords: epoché, Hilbert arithmetic, Husserl reductions, information and quantum information, qubit Hulbert space

I INTRODUCTION ENTERING FROM PHILOSOPHY

Husserl's phenomenology suggests a reinterpretation of the previous tradition (which is that of Western philosophy starting from Descartes) as any other great philosophical doctrine:

The mind - body problem of Cartesian dualism was resolved only by Kant's transcendentalism, for that involving (rather implicitly) the concept of the totality, which was reinterpreted very soon by Hegel's dialectic of the "objective development of the Spirit" (where "Spirit" is what substitutes the totality).

One may suggest that phenomenology exploits the same core of the "totality" implemented successfully by the classical German philosophical discourse, applying it to new areas and fundamental problems by relevant terminology, which Husserl introduced. Then, "epoché" can be seen as the relevant "bridge" from Hegel's "synthesis" (and "triad", respectively) to Husserl's "reductions" of a few kinds, and then, even before "triad": from the implicit formal structure, which transcendentalism introduces for the "totality" meaning that necessary reversible doubling of the "immanent" and "transcendent" into the "transcendental"[1].

One can find the same structure of the totality, implying be doubled definitively, in Husserl's "epoché" if the "real" is what substitutes the "immanent"; the "unreal", the "transcendent"; and the "phenomenal" (or "reel"), the "transcendental". "Epoché" is that operation both mental and fundamental, able to deliver the "phenomenon" in Husserl's sense, which is essentially different from that of Kant, after whom it was rather linked to the "immanent" and opposed to the "transcendent".

If one prefers Hegel's vocabulary, the same essence would be expressed by the "triad" of "synthesis", "antithesis", and "synthesis", correspondingly.

Formal philosophy following the track of Husserl's "philosophy as a rigorous science" (implicitly embodying that paradigm also implemented in mathematics) is to abandon the question of which the relevant interpretation is among the enumerated above (or other similar) ones once the analogical question of reality has be forsaken or "bracketed" definitively as to "epoché":

One is to restrict the research to the *formal structure of doubling* implied by the totality, maybe surprisingly identifiable as that of *a bit of information*. Indeed, if the visualization of a bit of information by a tape cell of Turing machine is used, the "phenomenon" (respectively, "synthesis" or "transcendental") corresponds to an empty cell, "reality" ("thesis", the "immanent"), to "1" (or to "0", the one alternative), and "unreality" ("antithesis", the "transcendent"), to "0" (or to "1", the other alternative).

Further, that formal understanding of "epoché" is the sufficient basis for the formal reinterpretation of all Husserl reductions, namely eidetic, phenomenological, psychological, and transcendental, by the concept of information applicable as to finite sets or series as to infinite ones (properly, quantum information in a preliminary understanding opposing it disjunctively to "classical" information as "infinity" to "finiteness").

II INTRODUCTION ENTERING FROM MATHEMATICS

One can grant that the philosophical problem of the totality implying definitively doubling shares the same formal structure as that about the foundations of mathematics (respectively, the corresponding "crisis" or "paradoxes"): that is the structure of a bit of information as above. Indeed, that "completeness" would be to be interpreted as the unification of Husserl's "unreality" (which can be interpreted as the mathematical reality of all models only related to the physical and material reality by itself distinguished from it "either - or"), on the one hand, and his "reality of the natural attitude", on the other hand, by the concept of "phenomenon", which should corresponds to a Pythagorean identification of the "unreality" of mathematical reality with the physical reality by itself, after which the being in the most fundamental philosophical sense is mathematical in the final analysis.

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[1] The paper is based on his article for Encyclopaedia Britannica (1927) as the most concise source relevant for formalization. The investigation of the development of Husserl's ideas and concepts is far from its intention.

That is the corresponding formal structure of a bit of information, implied by the totality, but interpreted now particularly to the foundations of mathematics or its philosophy (necessarily turning out to be Pythagorean), is representable by the opposition of all mathematical models ("0", the one alternative) to the world by itself ("1", the other alternative), and both identifiable as the same (e.g. as in the visualization by an "empty" tape cell of Turing machine) therefore overcoming Descartes's dualism simply postulating for the "body" and "mind" to be the same not less than being two absolutely distinguished entities.

Two of the most famous and relevant synoptic demonstrations of the troubles about the consistent completeness of mathematics are Russell's paradox about the set of all sets (Russell 1902) and the Gödel incompleteness theorems (1931). Therefore, they can serve for representing concisely the way, in which the above "Pythagorean solution" for mathematics can be established:

Russell's paradox reveals the well-known unresolvable antinomy about the set of all sets not belonging to themselves, whether it belongs to itself or not. The Zermelo (1908) solution (at that, being *ad hoc*) does not allow for the set of all sets to be admissible (e.g. postulating the condition for any admissible set to be a subset, and thus, the set of all sets does not satisfy it definitively).

The criticism of it consists in the observation that it is too strong, therefore excluding relevant and consistent theorems, methods, and even areas of mathematics, cancelling factually its completeness in order to avoid Russelll's direct corollary from Cantor's "naïve" set theory.

Once the totality is admitted as a *mathematica*l postulate, and thus as a *scientific* concept (Penchev 2020 August 25), Russell's paradox can be reinterpreted as a special (even definitive) property of the totality or as a corollary from it in the particular case of the "set of all sets" (now understood as an embodiment of the totality to be applicable to the foundations of mathematics):

Then, the "set of all sets" (as a particular expression for the totality) impling for it to be definitively doublable: once, as the set of all sets *belonging to themselves*; and twice, as the set of all sets *not belonging to themselves*. The reconciliation of Russell's paradox to the standard scientific propositional logic needing the "law of noncontradiction" (therefore allowing for that antimony to be interpreted as a usual statement) consists in the following:

The formal structure of a bit of information is definitively (and thus necessarily) attached to it and unlike any part of the totality, to which the researcher's free will can choose to consider it as a wholeness attaching the bit at issue or not. That is, the researcher must choose: either to interpret the investigated entity as a wholeness by assigning the bit at issue (properly originating from the totality once postulated mathematically), or to interpret it as a part of any other whole, therefore excluding the bit in question:

Then, the totality is defined as that unique case where that bit is to be ascribed definitively to the entity, claiming to be the totality, and therefore suspending the researcher's choice to add it or not. That solution can be interpreted in relation as to Russell's paradox itself as to its "classical" solution restricting a set to be definiable after the condition to be a subset (of some other set, which is not necessary to be certain explicitly):

So, a bit of information, namely that of Russel's paradox, is assigned to the set of all sets, and its two opposite alternatives, which cannot be considered simultaneously according to the definition of a bit of information, are correspondingly: the set of all sets *belonging to themselves*, the set of all sets *not belonging to themselves*. The law of noncontradiction is obeyed, for Russell's antinomy is reduced formally to a "harmless" usual statement by dividing the two contradictory alternatives to those of the complementing bit.

Obviously, this is a *formal* solution consequently applicable to any antinomy of any kind; in turn it allows for a newly "paradoxical" definition of a bit of information (and thus, implying a reformulation of the quantity and concept of information as far as the bit of information is its unit, by which it can be measured): *a bit of information is the minimal complement (or solution) of any antinomy to a consistent statement*.

As to the classical solution of Russell's paradox, it is isomorphic to the addition of a complementing bit of information as above. Indeed, that bit is: a certain element belongs or not to any set at issue. Thus, the set of all sets, not satisfying that condition, is not a "set" after the restriction and Russell's paradox does not make sense. Indeed, this solution is too strong, quite redundantly excluding the set of all sets (and thus, the totality from mathematics) and preventing any eventual future construction of a bridge able to smoothly connect mathematics to philosophy, physics, and all branches of science in the final analysis.

On the contrary, the set of all sets can be consistently admitted if a complementing bit formal and absolutely independent of any eventual interpretation is necessary in its definition originating from that of the totality as it is elucidated above. One can track that the same complementing bit of information can remove the Gödel incompleteness of arithmetic to set theory:

Concisely, the Gödel incompleteness theorems (1931) mean that Peano arithmetic is either incomplete or contradictory to set theory (ZFC). One can reformulate that result by investigating the necessary and sufficient condition, under which a system including Peano arithmetic and (ZFC) set theory as a true subclass would be both complete and consistent. In other words, one is to search for and then research the complement (in the sense of set theory) of the set corresponding to the former class, to the set corresponding to the latter class.

A heuristic direction how the complement at issue can be reached in an explicit and preferably constructive way can be suggested by the solution of Russell's paradox as far a one can find a certain formal similarity to the antinomy of the Liar utilized by Gödel himself in the introduction of his paper (Gödel 1931: 175-176) to explain the intention, sense, and scheme of the proof based on the existence of the *finite* Gödel number attachable to the proposition stating to be false (analogically to the "Liar"); even more so that again Gödel himself has written in a footnote, "14" (Gödel 1931: 175) that *any* antinomy would be relevant to be used.

Russell's paradox, that of the Liar, and the Godel proposition stating to be false share the deficiency of a bit of information, after the eventual addition of which all the three are transformed in "harmless" sentences as statements whether true or false. That missing bit of information in relation to Russell's paradox is elucidated above.

As to the "Liar", one can interpret that bit so that its alternatives are: (1) the Liar's statement refers to itself therefore being true; (2) the Liar's statement refers to anything else therefore being false. In fact, the construction "I lie" is elliptic, and the real use always means explicitly or implicitly the complete sentence "I lie that (any proposition relevant to me) " and furthermore restricted to (2): thus adding the bit at issue by its value exactly determined as (2). The elliptic construction "I lie" is similar to that of the "set of all sets" due to a missing bit of information[2].

As to the Gödel proposition stating to be false (respectively, by means of its Gödel number), the alternatives of the missing bit can be determined so: (1) the proposition stating for itself to be false is true; (2) it stating for anything else to be false is false. Then, the problem is whether a certain *finite* Gödel number can be assigned unambiguously to the Gödel proposition once it is determined only to the "nearest bit of information". In other words, its implicit and binary uncertainty consists in the option to be interpreted in two ways inconsistent to each other: (1) as an element of the class; (2) as that class itself; or respectively, (1) as an element of an implicit class; (2) as the same element considered as class (for example, as in Russell's theory of types e.g. according to his "Principles of mathematics"[3]) distinguishes them fundamentally and forbids them to be used simultaneously, and the Gödel proposition does just this, therefore violating the prohibition.

That binary uncertainty can be overcome if and only if two mismatching Gödel numbers are ascribed to the Gödel proposition. They can be identified with each other if and only if they are *infinite* ordinal numbers in virtue of sharing the same (infinite) cardinal number. Consequently, they cannot be *finite* natural numbers (as it seems initially) for all natural numbers are finite, due to the axiom of induction.

One can track that bit of uncertainty following Gödel's original proof, but only conceptually according to the philosophical "genre" of the present paper. It can be found in the Gödel proposition though remaining only implicit and therefore needing an elucidation to be noticed. The proposition states that the proposition possessing a certain Gödel number is false and that the certain Gödel number at issue is that of the proposition itself.

Following the track of the "Liar", the proposition restricts itself only to the "true half" of the Liar's statement: namely, that "I lie that I lie (that I say truth)" is a true statement in the virtue of the tautology that the Liar is a liar. However, that restriction is possible only at first glance, but implies just one bit of uncertainty, in fact, due to the inseparability of the "true half" from the "false half":

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[2] And vice versa: one can admit that the converse statement is also true. Namely, *any* non-paradoxical statement can be transformed into an antinomy by depriving it of a bit of information chosen relevantly and the existence of which is *necessary*. For example, "The moon is full" as far as this means both a sentence, which is "true" being built correctly (as in the case), and a fact being true if only if it is adequate to the experience. Without assigning that bit (eventually, by means of a certain value of the bit), the statement is a paradox, just as the "Liar". Indeed, the syntactically correct sentence interpreted as a fact implies the contradiction of being both true and false.

[3] In detail in: Daesuk, DePaul 2013; Rodriguez Consuegra 1989;

Indeed, after reflecting backwards, to itself, the proposition is to be represented so: the proposition possessing its Gödel number is false. Then, this is equivalent to the negation of the conjunction of the following two statements: (A) “The proposition possesses that Gödel number”, and (B) the proposition itself; so that: $\neg(A \wedge B) = \neg A \vee \neg B$. Further, the original proof postulates that $\neg A \vee \neg B = \neg B$, which is equivalent to “A is true”. However, this is an additional postulate justified only ostensibly by the “lemma” to it, namely that there exists a bijection of all natural numbers in Peano arithmetic and all propositions in the first order logic meant and defined by the Gödel paper (i.e. the “Gödel enumeration”).

Then, all those propositions of the *first order* logic cannot include the Gödel proposition if it *refers to itself*. Its transformation into a “normal” first order proposition by the mediation of the Gödel enumeration is nominal, only at a first glance. The reason is that the Gödel enumeration lemma includes as a condition *all natural numbers in Peano arithmetic* (in which all natural numbers are finite in virtue of the axiom of induction), the enumeration of the Gödel proposition needs the ***set*** *of all natural numbers in Peano arithmetic* (which is infinite in virtue of the axiom of infinity, e.g. in ZFC).

Then, the Gödel number of the Gödel proposition is either infinite (transfinite) or it is ambiguous, being finite, if the postulate "$\neg A \vee \neg B = \neg B$” above is not accepted, the statement A is false, and the real Gödel number of the Gödel proposition is some other and different from that only alleged in A. In other words, the implicit bit of uncertainty in relation to the Gödel proposition in the framework of Gödel’s original proof is due to the opposition “the Gödel number of the Gödel proposition is an element of the set of all natural numbers in Peano arithmetic” versus “the Gödel number of the Gödel proposition is a natural number in Peano arithmetic”. The mismatch of the two interpretations does not appear in any proposition, which does not refer to itself indirectly by the mediation of its own Gödel number. The same opposition can be developed further (as far as any bit is two oppositions: a) “no choice” versus “choice”; b) if the case is “choice”, the “one alternative chosen” versus the “other alternative chosen”). If the “Gödel number of the Gödel proposition is a natural number in Peano arithmetic” is the case, “the explicit number in the Gödel proposition is true” versus “it is false” (i.e. the real number is certain, but other than that alleged in the proposition and unknown).

That consideration makes clear that Gödel’s proof relies on a missing bit of information just as Russell’s paradox does, but does not offer yet any construction able to complement both Peano arithmetic and set theory to a complete and consistent system. Anyway, the discussion now suggests that the bit is to be addable only at the metalevel since the adoption of a bit to Peano arithmetic would remain it just the same (i.e. identical).

Hilbert arithmetic debated in detail in other papers (e.g.: in Penchev 2020 August 25) is an example of how Peano arithmetic can be complemented in a way to be both complete and consistent with set theory. Furthermore, Hilbert arithmetic can be considered as complementary or equivalent to the qubit Hilbert space (and thus to the separable complex Hilbert space of quantum mechanics in the final analysis) so that it only “translates” the existent proof of the absence of hidden variables in quantum mechanics (Neumann 1932; Kochen, Specker 1967) into the "language of mathematics and its foundations" (i.e. both arithmetic and set theory).

Indeed, Hilbert arithmetic can be seen as Peano arithmetic, to which only a single bit is added, but as if "outside", i.e. at its metalevel so that it is able to consistently include set theory as a true part. Peano arithmetic is doubled by its isomorphic, but anti-isometric counterpart needing exactly two substitutions starting from the former, "normal" Peano arithmetic: (1) it begins from "infinity" (i.e from the less *countable* ordinal often notated by "ω")[4]: it substitutes "1" of the "normal" Peano arithmetic; (2) the function successor is defined as "$n - 1$"rather than as "$n + 1$".

Speaking loosely, the second "twin" of Peano arithmetic starts from "infinity" moving backwards to the finiteness where the "first twin dwells" as far as it starts from "1" moving "forwards" to infinity. The two twins are indistinguishable from each other in all the rest and can be granted to be identical as two "complementary" copies of the same. They can be interpreted equally well as the two alternative elementary choices of a bit of information identifiable as the bit which is to be added to Peano arithmetic "externally".

The last statement implies the hypothesis of the existence of a "coherent state of both twins, preceding each of them". It can be represented by "mixing" the two oppositely directed well-orderings e.g. as if each well-ordering is interpreted by the binary relation " $\geq$ ", " $\leq$ " correspondingly rather than by " $>$ ", " $<$ ", after which the coherent state turns out to be a colossal equality "$1 = 2 = 3 = \ldots = \ldots = \omega - 2 = \omega - 1 = \omega$".

It can be interpreted as a special, nonstandard or "degenerated" Peano arithmetic, which can start from any element or "natural number", and the function successor is defined as "$n =$ " as far as any unary operation implies a certain binary relation of the operand and result. One can prove that the nonstandard Peano arithmetic possesses the structure of Boolean algebra (after a relevant definition of the Boolean operations), by which it can be identified as isomorphic to both propositional logic and set theory.

One can notice that Hilbert arithmetic corresponds exactly to the qubit Hilbert space as follows. The former can be acquired from the latter as the class of equivalence of all possible values for each qubit. Conversely, the latter follows from the former after the unambiguous distinction of any element within each class of equivalence, which any unit of Hilbert arithmetic is. This allows for the identification of Peano arithmetic with any qubit, as repeated locally within each unit of itself as the one or other counterpart of Peano arithmetics belonging to Hilbert arithmetic.

Those exact (in the two directions) mappings of Hilbert arithmetic and the qubit Hilbert space are the "smooth bridge" between physics and mathematics, necessary (but not sufficient) to

[4] The least countable ordinal number exists in virtue of the axiom of choice able to order well the set of all countable ordinals. So, the axiom of choice as a relevant equivalent is necessary in Hilbert arithmetic to settle the relation of the two counterparts of Peano arithmetic. The sense of the axiom of choice in Hilbert arithmetic consists in postulating that "metabit" in question, i.e. the bit of the next metalevel, also interpretable as a "primary choice" for either the one or the other "twin" to be chosen initially.

establish Husserl's phenomenology as a rigorous science[5] (in an absolute literary and exact meaning rather than as a metaphor). One is to complement it by the philosophical interpretation of the bit of information, which is to be added at the next metalevel, already implemented in advance in the philosophical introduction of the paper (Section I).

III INTRODUCTION ENTERING FROM PHYSICS

It needs the exhaustive interpretation and thus unification of quantum mechanics and the theory of quantum information, often denoted briefly as "quantum mechanics and information" undertaken in the last quarter of the 20th century and underlain by the debate of Bohr and Einstein about the foundations of quantum mechanics, the proof of Bell's inequalities, the experimental confirmation of their violations, the investigation of various phenomena of entanglement, etc.

However, the only necessary result is the identification[6] of the qubit Hilbert space with the separable complex Hilbert space sufficient for quantum mechanics to be justified rigorously (Neumann 1932): after that the foundations of quantum mechanics (namely, the separable complex Hilbert space) and mathematics (namely, Hilbert arithmetic) are connected stably, reliably, and smoothly to each other. Any physical result can be interpreted as a mathematical theorem in the final analysis and not less: vice versa.

Indeed, modern physics appeared as mathematical after Galileo and Newton. A bewilderment ascribed to Einstein[7] was why the world is rational. Indeed, the rationality of the world allows for physics and science to occur and establish, but itself seems to be a strange, unexplainable, and confusing fact, a ridiculous "gift of God" without any natural cause or simply any reason.

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[5] Obviously the allusion is to the famous paper of Husserl (1910): its context is also necessary to formulate the way in which "phenomenology as a rigorous science" can be considered literally rather than metaphorically.

[6] That identification is almost trivial e.g. after the following convention. Any vector belonging to the separable complex Hilbert space implies *unambiguously* a vector, the components of which are qubits, the number of which are less than the former just by a unit. Accordingly, unitarity is modified quite weakly by virtue of that unit. If one grants the separable complex Hilbert space to be infinitely dimensional in general, it is identical to the qubit Hilbert space. Any finite-dimensional subspace of the separable complex Hilbert space can be equivalently represented as infinitely dimensional by adding an infinite number of zero coefficients.

[7] The sentence ascribed to Einstein is: "The most incomprehensible thing about the world is that it is comprehensible" (e.g., https://www.brainyquote.com/quotes/albert_einstein_125369). In fact, the real words of Einstein (1936: 351) are quite loosely interpreted and then ascribed to him as an exact quotation. Einstein wrote literally: "One may say "the eternal mystery of the world is its comprehensibility." It is one of the great realisations of Immanuel Kant that the setting up of a real external world would be senseless without this comprehensibility." Einstein's conclusion is the following: "It is in this sense that the world of our sense experiences is comprehensible. The fact that it is comprehensible is a miracle." Anyway, that "Einstein's sentence" loosely retold and presented as a citation is a cultural artefact. (The translation in English by J. Picard is cited here: Einstein's original in German is also published in the same issue of the journal immediately before the translation: pp. 313-245)

If the foundations of physics and mathematics can be merged after both Hilbert arithmetic and qubit Hilbert space (respectively, the separable complex Hilbert space) are complementary to each other, one can interpret the mathematical "half of the world" as that implicit, but absolutely necessary "half" complementing the experienceable "half" of the real physical experiment at the moment therefore delivering the rationality of the world and so astounding for Einstein.

Adopting the fundamental randomness of any quantum entity, humankind accesses the absolute rationality of the world. This is a statement only seeming (only at a first glance) to be paradoxical: both fundamental randomness and absolute rationality are the two alternatives of a bit of information, being in a coherent state in the world by itself.

Husserl's phenomenology (following the track of philosophical tradition) copies the same formal structure of a bit of information, and moreover, it shares a similar interpretation as the motivation for "philosophy as a rigorous science",

That "philosophical" bit of information can be visualized by the missing two bits after (quantum)[8] teleportation as well as due to merging the foundations of physics, mathematics, and philosophy. The two bits are to be delivered by any subliminal channel rather than quantum or instantly. They are two oppositions after the standard definition of a bit of information and neglecting an implicit preliminary opposition between the absence of choice (an empty tape cell of Turing machine) and the choice itself, in turn divided explicitly into the two alternatives (denoted usually as "0" and "1" in relation to the Turing machine tape cell). So, one can conclude (for an eloquent paradox) that *any bit consists of two bits*, in fact, because of the explanation just above.

So, the two deficient bits (equivalent to a single bit) after teleportation are isomorphic to the "philosophical" bit of information. Furthermore, the missing two bits e.g. after transmitting or better, exchanging of a qubit of quantum information between Alec and Barbara (or Alice and Bob) can be interpreted philosophically enough as follows: (1) the coherent quantum superposition of Alec and Barbara versus their decoherent states as properly Barbara and Alec in two space-time points arbitrarily distant from each other; (2) the choice of whose experience: either Barbara's or Alec's.

So, if one compares the two exchanged qubits (rather than only one transmitted from Alice to Bob) they would be the same as in (1) to (2) as in Alec's experience to Barbara's. Then, the philosophical bit of information, but already in terms of Barbara and Alec's experiment, can be interpreted as necessary to distinguish Alec from Barbara unambiguously and ultimately: (1) Barbara and Alec are different entities; (2) if the case is (1), this is Alec, and this is Barbara.

As a formal structure of the experiment, only a single qubit is sufficient, but tripled for the experiment to be described exhaustively. Just the tripling into a triad of identical *qubits* is the physical sense and meaning of the "philosophical" *bit*[9] of information. Conversely, the triad of Husserl's epoche can be described perfectly formally by the symmetries of three identical qubits,

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[8] The brackets serve to hint that any teleportation different from quantum does not exist.

[9] One can notice that the bit of three identical qubits is isomorphic to the qubit Hilbert space due to the axiom of choice.

however interpreted in a certain Husserlian manner: e.g. as an arbitrary entity, its mental image, and their shared phenomenon identifying the former two in virtue of "epoché".

IV HUSSERL'S PHENOMENOLOGY INTERPRETED BOTH FORMALLY AND AS THE UNIFICATION OF PHILOSOPHY, PHYSICS, AND MATHEMATICS

One needs what "formal interpretation" means after a rigorous interpretation of the term: a certain mathematical structure is assigned to Husserl's phenomenology as a philosophical doctrine and claiming to be its exhaustive model just as mathematical structures can be models of physical theories. The same structure is to be shared by theories referring to the foundations of physics (quantum mechanics and information), mathematics (Hilbert arithmetic), and philosophy (Husserl's phenomenology) therefore unifying and merging them.

That enterprise seems to be eclectic and foredoomed, but only at a first glance. The prejudice against it, rooted in both philosophical and scientific traditions, can be rationalized into the following two main objections: (1) their subjects are not only absolutely different, but incompatible and incommensurable; (2) not only their approaches and methods are absolutely different, but even they confess incompatible and incommensurable concepts of truth: speaking loosely, logical consistency and completeness for mathematics; experimental confirmations of competing mathematical models (therefore consistent and complete logically in advance) for physics; the dogmatic and fundamentally unfalsifiable postulation for philosophical doctrines anyway competing between each other for proponents and supporters just as any other artefact in art, humanity, and politics.

Nonetheless, those redlines of distinction might be overcome under certain conditions partly available in the philosophical and scientific tradition of Modernity until now. The gap between physics and mathematics can be bridged by a theory interpretable both mathematical and physical containing an internal proof of its consistent completeness for mathematics and confirmed by the experiments up to now for physics, furthermore making testable and thus falsifiable predictions.

The gap between the unfalsifiable philosophical doctrine and a scientific theory, whether physical or mathematical, being falsifiable definitively though following different criteria for falsifiability can be cancelled by a kind of "scientific transcendentalism"[10] sketched here as well as in other papers (Penchev 2021 April 12; 2021 February 25; 2020 October 19). It relies on the postulate of the totality featuring philosophical transcendentalism, but interpreted now as a scientific principle thus allowing for its estimation according to conclusions, direct corollaries, and general consistency or compatibility with other theories in physics and mathematics (first of all, referring to their foundations).

Husserl's idea for "philosophy as a rigorous science" coincides with that of scientific transcendentalism in a few essential features whether explicitly or implicitly. He followed the traditions of both mathematics (being a mathematician by education) and philosophical

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[10] Particularly, "mathematical transcendentalism" or "physical transcendentalism", "physical and mathematical transcendentalism".

transcendentalism (suggesting for "phenomenon" to be definiable by "epoché to reality" (i.e. to the reality of any entity) furthermore searching for the general logical and transcendental foundations shareable as by arithmetic as by any science striving or claiming to be strict and rigorous (for example, meaning even so humanitarian disciplines as philosophy or psychology researching Spirit or mind and seeming to be too elusive and self-contradictory, at least in comparison with the subjects of mathematics or physics).

One can demonstrate that the mathematical structure of Hilbert arithmetic, furthermore identifiable as the structure dual (and equivalent in the sense of complementarity) to the qubit Hilbert space, can claim to satisfy the requirement notated by (1) above. As to (2), one is to infer Hilbert arithmetic (a.k.a. the qubit Hilbert space of quantum information, a.k.a. the separable complex Hilbert space of quantum mechanics) from the scientific postulate of the "totality". As far as this has been done in other papers (e.g. in *Penchev 2020 October 20*) more or less, the clear demarcation of physical, mathematical and especially philosophical parts of the ultimately assembled, integrated and fitted structure can be the specific accent in this paper, moreover it assists to be reinterpreted Husserl's reduction and even, to be found their counterparts in fundamental physical symmetries (such as the symmetries U(1), SU(2), SU(3) of the Standard model, the symmetry of local and global space, etc) or in fundamental mathematical symmetries (e.g. the symmetry of finite and infinite sets after Skolem's "relativity of the concept of set", the symmetry of idempotency, such as that of the two dual Hilbert space, etc).

The qubit Hilbert space can be identified with Hilbert arithmetic as it is described above, but also by the following procedure. All elements of the qubit Hilbert space (speaking loosely, also all qubits, or all wave functions or all quantum states in the universe possible ever) can be ordered well in virtue of the axiom of choice and thus mapped bijectively into all transfinite natural ordinal numbers less than the least infinite countable ordinal number.

Then, they can be interpreted also as all finite natural numbers of the twin (dual) Peano arithmetic, but as if "seen" from the viewpoint of the other twin: as following from its beginning by its function successor. Properly, the relation of idempotency between the two dual Peano arithmetics (each of which seems as the qubit Hilbert space from the "viewpoint of the other one") corresponds isomorphically to the Heisenberg uncertainty after one has identified the certainty of a quantum state to be measured quite exactly:

Indeed, the cost of that absolute accuracy in quantum mechanics (unlike classical physics where the analogical absolute accuracy is "for free") is just the absolute uncertainty of the dual wave function (i.e. the wave function of the conjugate quantum quantity or state) in turn being mathematically the same as "empty qubits" or the "classes of equivalence of all possible values" of those qubits: i.e. in the final analysis the units of the usual Peano arithmetic.

Summarizing concisely, the physical sense (but not less: the strict meaning) of Peano arithmetic (and thus, of arithmetic at all and then, of mathematics) is to be interpreted as the conjugate counterpart of the physical world being complementary to each other and also equivalent though in the sense of Bohr's complementarity and as far as it allows for complementary entities to be considered as equivalent (i.e. as a consistent metaphysical assumption unfalsifiable and excluding fundamentally any experimental verification). Anyway, that complementarity can be transformed into falsifiable by virtue of “scientific transcendentalism”: namely, investigating its logical consistency in Hilbert arithmetic and demonstrating its interpretability as the physical world of quantum mechanics.

Meaning the above consideration, the “philosophical” bit of information would consist of the following (or better, its sense consists *in* the following): (1) the wholeness (corresponding to the “totality”) of all natural numbers as an *infinite* set (though all natural numbers are *finite* in Peano arithmetic by virtue of the axiom of induction) is added externally to Peano arithmetic, but (2) only as an option. i.e. formally representable as a bit of information; (3) that option applied to itself (“self-referentially”) implies the identification of the “empty bit, the one and the other of its alternatives” (all the three) as three copies of the same qubit; one can notice that (4) the structure meant in (3) is isomorphic to the qubit Hilbert space due to the axiom of choice; at last consequently, (5) the “philosophical” bit of information is both complement of all structures studied by mathematics to the physical world investigated by quantum mechanics, on the one hand, and identification of them as the same qubit, on the other nand. This means: the “philosophical” bit of information (1) introduces and thus distinguishes the local from the global space by a bit of classical information, and (2) allows for them to be identified, after that, as the same qubit of quantum information.

That “global bit” consisting of three identical “local qubits” at the corresponding three sublevels is the formal structure granted to be relevant to phenomenology. The isomorphism of it to both Hilbert arithmetic and qubit Hilbert space has been already demonstrated. What follows is the interpretation in the proper terms of Hussel’s phenomenology.

V HUSSERL’S PHENOMENOLOGY AS A RIGOROUS SCIENCE

A few preliminary notices are to elucidate:

(1) The several kinds of *reduction* suggest implicitly to be unfiable as the same reduction appearble differently, namely as eidetic, phenomenological (psychological), or transcendental reduction, each of which emphasizes one certain different aspect of that general reduction therefore neglecting the rest ones. So, phenomenological (psychologic[11]) reduction accents on the *symmetry*[12] of reality (or Descrte’s “body”) and its mental correlate, or the pure phenomena of the things themselves; eidetic reduction, to the symmetry of the single wholeness of any entity, e.g. claiming to be mental, and the infinite variety of its properties or accidental features e.g. claiming to be “real”; at last, transcendental symmetry able to unify and identity the former two

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[11] Respectively, psychological reduction emphasizes a certain sub-aspect of phenomenological reduction, after which phenomenology can be considered as a doctrine of pure psychology purified from the accidental impurities of psychologism to its necessary essence. It can be interpreted as a result of the psychological counterpart of “epoché”, which abandons whether an entity is mental or not, analogically to “whether it is real or not”. In other words, both phenomenological and psychological reduction share the same formal structure, which is that of the “philosophical” bit of information.

[12] The term “symmetry” is not relevant to the authentic Husserl doctrine not being used explicitly in his papers, but it is even necessary if the objectivity (as here) is to reveal a hidden underlying formal structure able to unify phenomenology with mathematical and physical theories.

can be accepted to be a meta-symmetry of the phenomenological and eidetic reduction implicitly defining the totality or wholeness being valid to any wholeness or completeness, but not to any part thus neither "whole", nor "complete".

(2) The concept of symmetry is meant implicitly by Husserl in the framework of his terminology of "epoché", "phenomenon", "reduction", etc. The sense of "symmetry" in the context of Husserl's phenomenology is to be understood as follows. Fundamental philosophical distinctions such as those of Descrtes's dualism or the opposition "subject - object" in classical German philosophy are underlain from their symmetry. The same idea in the present paper is necessary to reveal the fundamental structure of the "philosophical" bit after which the symmetry corresponds to the state before the distinction and which makes the latter possible.

(3) At the same time, the concept of symmetry allows for phenomenology once considered formally to be unified with mathematical and physical theories, in the foundations of which is used universally rather than only often. For example, the concept of symmetry distinguishes and unifies the class of equivalence (what "set" means particularly) with each of its elements in mathematics; or the three fundamental physical interactions of the Standard model: the weak SU(2), the strong SU(3), or the electromagnetic U(1) interaction.

Meaning the above preliminary notices, the explication of Huserl's phenomenology as a rigorous science can be reduced to: (a) the symmetries of classical and quantum information embedded into the structure of a bit consisting of three identical "empty" qubits, already discussed in another paper (Penchev 2021 June 8); (b) the interpretation of all Husserl reductions by the same structure, which follows in detail now:

As to *phenomenological (psychological) reduction*, one is to mean the symmetry of the two alternatives of a bit of information (such as the "philosophical" bit of information), both excluding the symmetries of the bit corresponding to phenomenon with each of both alternatives in virtue of time arrow granted to be rather a fundamental condition in Husserl's phenomenology. Anyway, one can admit a kind of generalized phenomenology, in the framework of which time is suspended thus allowing for the other two symmetries as possible counterparts of phenomenological reduction.

As to *eidetic reduction*, one is to mean the symmetry of an empty qubit as the class of equivalence of all possible values of it, "locally" to its wholeness, "globally", i.e. in the structure of the bit at the metalevel. In other words, eidetic reduction corresponds to the symmetry of the philosophical metalevel to the scientific level both physical and mathematical and thus to the idea of scientific transcendentalism symmetric to philosophical transcendentalism. Indeed, Husserl himself explained eidetic reduction by the class of equivalence of the eidetic core to all possible properties of the entity at issue, an infinite set presumably in general. Also, eidetic reduction can be understood as the identification of the global and local space in the Standard model.

As for *transcendental reduction*, the great philosopher suggested that it was the same as phenomenological reduction, but interpreted to the totality itself. One can admit that transcendental reduction is able to unify phenomenological and eidetic reduction as merging into each other in virtue of the postulate of the totality: namely, once doubled by itself and embedded in the concept of phenomenological reduction, and twice, self-referential also by itself in eidetic reduction, Then, the totality itself can be defined exhaustively by the above identification of both “phenomenon” and “eidos” of anything.

Though the postulate of the totality was formulated initially in philosophy as speculative, non-empirical, metaphysical, and unfalsifiable, scientific transcendentalism constitutes itself by its transfer in mathematics and physics and its translation into their language(s) therefore admitting the double verification of logical completeness and consistency, and experimental confirmation for its corollaries and conclusion, the refusal of which would reject the postulate itself in virtue of *modus tollens*.

Most of the testable statements (whether logically or experimentally) are constructed by the concept of the wholeness of anything, meaning that any entity can be considered as a *particular totality* in the following rigorous meaning. As “particular”, it is to be a part of something whether explicitly or constructively known or only necessarily existent. Nonetheless as a wholeness, it should be similar to the totality, doubling itself transcendentally in two coinciding hypostases, both as “phenomenon” and as “eidos”, different from each other in the former consideration as a part from something.

Those two viewpoints to any entity do not generate a contradiction being complementary to each other, forcing the researcher to choose in advance only one of them. Furthermore, one can notice that those two viewpoints (to anything) share the same formal structure of a bit of information, therefore being identifiable as the “philosophical” bit of information.

Moreover, the aspect of the mismatch of eidos and phenomenon can serve further as for the philosophical definition of motion (or synonymically, “movement’) as the necessary (and thus essential) property of whatever claiming to be physical. A sufficient, but not necessary condition of that non-coincidence is time, or respectively, well-ordering. On the contrary, the complementary mathematical aspect to anything (particularly implying general mathematizabilty: *Penchev 2020 June 29*) consists in the identification of phenomenon and eidos, implementable in a relevant mathematical “model”[13] and originating directly from the postulate of the totality. Consequently, the totality is to be definitively mathematical rather than physical, empirical or experimental: an observation reflectable as Pythagoreanism, or rather, as neo-Pythagoreanism.

At last, one can track how the formal structure of a bit consisting of three identical qubits is relevant to transcendental reduction as well as to its interpretation by the identification of phenomenological and eidetic reduction. One considers the infinite set of bits embeddable in each bit internally or locally, i.e as a qubit in the final analysis (Penchev 2021 July 10). The procedure generates again[14] the qubit Hilbert space as complementary (or identical) to Hilbert arithmetic.

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[13] The quotation marks serve to hint that the “model” at issue coincides with reality, unlike the usual use of the same term in default.

[14] Another method was used above.

The axiom of choice allows for the identification of any triple of "number-sake" (namesake) qubits with the complete qubit Hilbert space since the countable set consisting of countable sets is in turn countable in relation to all elements of the latter sets. Just that identification corresponds to transcendental reduction by means of the identification of phenomenological and eidetic reduction.

The main conclusion of the present section can be that (1) Husserl's phenomenology granted to be formalizable describes in detail the "philosophical" bit of information; (2) it both unifies and distinguishes as complementary the physical and mathematical aspect of the world, once interpreted formally.

VI A TEST BY HILBERT ARITHMETIC

Husserl (1900-1901) changed his viewpoint to the foundation of arithmetic radically in "Logical investigations" in comparison with his previous work (1891). He tried to justify arithmetic in the framework of psychologism, i.e inferring it from human experience, eventually mental in an inductive way in the sense of "physical induction". The subsequent root change as a result of the reflection and reinterpretation of his initial intention reordered logic, arithmetic, and psychology just in that sequence of fundamentality therefore allowing for psychology (and philosophy, in the final analysis) to be justified logically, i.e as a "rigorous science".

That general idea of "Logical investigations" partly corresponds to the almost contemporary to it, large-scale work in three volumes, "Principia mathematica" undertaken by Russell and Whitehead (any edition) to ground all mathematics rigorously only on a logical foundation. So, both share the idea, arithmetic to be inferred from logic, though the discourse of the former is rather philosophical, and that of the latter is mathematical, but Husserl, turning "upside down" his initial intention (1891), continued further, to the deduction of psychology and philosophy on the same logical basis, thus as a unified rigorous science of both philosophical Spirit and human mind originating from the totality in the final analysis necessarily and logically.

The present paper shares the same "pathos", only articulating it as "scientific transcendentalism" and extending it to physics and the aspect of the world studied by it.

The most notorious fundamental result in the track of "Principia mathematica" are the Gödel incompleteness theorems (1931), already discussed above partly. If one interprets the "philosophical" bit of information according to the explicated by Gödel dichotomy in the relation of (Peano) arithmetic to (ZFC) set theory: either incompleteness or inconsistency, the former can be understood as a solution of the latter, at that, implying a complement of arithmetic to set theory consistent to both and overcoming the "incompleteness of arithmetic" by the unified structure of "Hilbert arithmetic":

The methodological essence of the solution consists in the addition of the dichotomy itself, demonstrated by Gödel, to arithmetic in order to become consistent to set theory for both are necessary for the consistent and complete foundations of mathematics. Indeed, any dichotomy implies a bit of information after its two alternatives are reliably divided as complementary to each other. The dichotomy turns out to be a contradiction (i.e. a violation of the noncontradiction law of propositional logic) if and only if the alternatives are necessary to be meant *simultaneously*:

The formal structure of a bit of information prevents the direct conflict by adding two oppositions, which can be interpreted by "terms of contradiction" as follows: (1) the contradiction is opposed to its solution; (2) the alternative of the solution is opposed to the other one. Properly, the formal structure of Bohr's complementarity is the same unlike Hegel's logical and ontological "triad" as far as it can be interpreted to identify the contradiction with the solution as "synthesis", however not less distinguishing them in "philosophical time" considering the "synthesis" as the new "thesis" at the next *meta*level[15].

Though quite meaningful, Hegel's approach is to be rather modified[16] to be identifiable with Bohr's complementarity or with any structure admissible by propositional logic as conscient to it. As it is well-known, it, applied directly, implies the rejection of the non-contradictory law of propositional logic. On the contrary, the additional bit of information is the general method for any antinomy to be reconciled with propositional logic by dividing the mutually exclusive alternatives of the contradiction to be valid in two mutually exclusive relations relevant for each of the alternatives correspondingly.

Returning to the Gödel dichotomy, one only is to apply the general solution of any logical paradox to the relation of arithmetic and set theory, more precisely, to the alternative of inconsistent completeness, by an additional bit of information and interpreting it in terms of the relation. Thus, it can be added only at the next metalevel: therefore doubling Peano arithmetic by its dual counterpart, "from infinity backwards", and unifying both "twins" cancelling their opposed well-orderings by the nonstandard Peano arithmetic and demonstrating that it shares the structure of Boolean algebra with both propositional logic and set theory.

One suggests that the essence of logic and set theory (at least as to arithmetic) is their isomorphism as the same Boolean algebra rather than their questionless absolute distinction to two quite different areas: logic and mathematics. Then, the nonstandard Peano arithmetic can bring the isomorphism to them as Boolean algebra is sufficient to be identifiable with them.

On the other hand (i.e. on the other side of the logical "bridge" which the structure of Boolean algebra builds between set theory and arithmetic), one is to make Boolean algebra clear as a modified, modulo 2 arithmetic[17] therefore unifying all odd natural numbers as the same and all even natural numbers as the same. In other words, the nonstandard Peano arithmetic cancels and replaces the well-ordering (definitive for Peano arithmetic due to function successor) by

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[15] By the way, a solution claimed by Marx to be his innovation and contribution to Hegel's original doctrine.

[16] For example, by the two philosophical doctrines utilized by Bohr to justify quantum complementarity, namely: Kirkegaard's "non-synthesizing dialectics" or the "Yin-Yang-Thao" structure of Chinese philosophy.

[17] If conjunction can be identified as the modulo 2 multiplication, the correspondence of disjunction and the modulo 2 addition though unambiguous needs a relevant redefinition; negation corresponds to the idempotent transformation of even and odd natural numbers. However, negation interpreted to the dual twin Peano arithmetics is to be identified with the correspondence of namesake natural numbers in each of them.

idempotency (furthemore, featuring a bit of information). Also vice versa: if modulo 2 arithmetic is given initially, one can obtain two Peano arithmetics only enumerating by natural numbers the consecutive idempotencies therefore distinguishing each of them unambiguously, i.e. adding a well-ordering in the final analysis[18].

Summarizing, Husserl's idea of logic underlying arithmetic is reduced formally in the present context as the structure of Boolean algebra (eventually interpreted as modulo 2 arithmetic) implying Peano arithmetic after removing the "philosophical" bit of information applied at the next metalevel and thus returning to the level itself[19].

VII CONCLUSION

The main conclusion consists in the observation that the advance of science needs the unification of philosophy, physics, and mathematics and especially, of their foundations. Until now, on the contrary, they are absolutely parted, confessing radically different conceptions of truth (correspondingly: metaphysical and unfalsifiable truth shared by promoters, companions, and followers; logical consistency; and experimental confirmation), and thus, in a state of permanent conflict for border and disputed territories.

The paper considers first of all the relevant changes in philosophy, which would allow for it to overcome the contradictions with mathematics and physics stepping on the pathway of unification. It means Husserl's phenomenology proclaimed for the first time philosophy to become a "rigorous science", moreover, following rather formal and logical ideas.

The testament and legacy of Husserl, often understood to be rather properly philosophical or even metaphorical, the paper interprets literally, namely, by discussing the necessary changes in philosophy by the case study of phenomenology if it wishes to be similar to mathematics and physics, i.e. indeed a "rigorous science". The conception of scientific transcendentalism intended as a falsifiable counterpart of philosophical transcendentalism is involved, and the postulate of the totality is to be formulated in a falsifiable way relevant to science.

Particularly, a certain mathematical structure inferable from the totality and shareable with the foundations of physics (quantum mechanics) and mathematics (arithmetic and set theory) is ascribed to phenomenology therefore conceding it to be absolutely formal against to the prejudice for philosophy to belong to humantarity, but just according to Husserl's insight if it is suggested to be applicable literally.

The structure at issue is that of the "philosophical" bit of information, unifying at the next metalevel, three identical qubits: thus identifiable as both the qubit Hilbert space (for the foundations of physics) and Hilbert arithmetic (for the foundations of mathematics. Special attention is paid to the Gödel incompleteness theorems (1931) because overcoming the restriction of them is a necessary condition for establishing that formal structure as general.

Finally, "epoché" and the phenomenological (psychological), eidetic, and transcendental reductions are reinterpreted formally, by means of that structure and reunified in virtue of it. A

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[18] One should notice that the equivalence of the axiom of choice and the well-ordering "theorem" is isomorphic to the relation of the nonstandard and standard Peano arithmetics.

[19] There exists a direct correspondence to Russell's theory of types.

new, formal and logical viewpoint to the relations and unity of the foundations of philosophy, physics, and mathematics is open. Speaking loosely, just the "philosophical" bit of information is sufficient to be added to (Peano) arithmetic, therefore transforming it into Hilbert arithmetic, to reconcile it with set theory as well as in order to identify it with the qubit Hilbert space whether as equivalent or as complementary; or quite metaphorically, but concisely: *philosophy unified with mathematics* (therefore implying a new form of Pythagoreanism) *is physics*.

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