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Study of the propagation of solitary waves produced by an assembly of quantum dots through optical fibers.

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Abstract. In this paper, the potential use of stacked layers of semiconducting nanostructures as optical field sources to optimize the propagation of pulses without losses along nonlinear optical fibers was studied. During this research, we propose the external excitation of stacked layers of semiconductor quantum dots SQDs through an optical source that allows the generation of solitonic waves that are propagated through an optical fiber with non-linear optical characteristics. Theoretically, the soliton formation is studied from the nonlinear interaction between the SQDs and the external optical excitation, considering it as a quantum system of three energy levels. In the study, the non-linear Schrödinger NLSE equation is solved numerically using the Fourier Split-Step method to understand the evolution of the soliton emitted by the SQDs inside an optical fiber with real physical parameters.

1. Introduction

The semiconductor quantum dots SQDs are semiconductor nanostructures that restrict the movement of charge carriers in the three spatial dimensions. Its discovery has motivated during the last two decades to carry out both theoretical and experimental research within the physics of the solid state and other related sciences such as optics. Some of the most recent investigations indicate that this type of heterostructures can undergo abrupt changes in the spectral response with minimal variations in their size and morphology, offering important applications to optics, among which are the lasers of new generations, diodes, light emitters, optical multiplexers, biosensors, spectral tuners, quantum computing, logic gates, among others [1-6].

Nowadays, it has been possible to combine nanostructures with other polymeric materials such as optical fibers, giving rise to nanocomposites, which are generally composed of several phases such as SiO₂, where one or several of its dimensions are found at the nanoscale [6-10]. The optical fibers are a important elements for the propagation of light waves with water and for the propagation of electromagnetic waves in the optical, infrared and ultraviolet range, however, there are other important applications for optical fibers in the area of sensors and in the study of nonlinear optical phenomena. In an optical fiber it is necessary to consider a theory of propagation of waves in dispersive media, in this sense, Schrödinger's nonlinear equation NLSE provides a complete description of a variety of localized non-linear effects that have been extensively studied in various contexts of the sciences and

that the theory can link directly with the propagation of intense optical pulses in non-linear optical fibers that give rise to the optical solitons. Although there are many experimental advances in the propagation of solitons in optical fibers, in practice the propagation of solitary pulses to large distances at a commercial level has not been possible due to the different technical functions [11,12]. The study allows us to propose a general model that allows coupling the solitons from the set of quantum dots with specific characteristics in an optical fiber with non-linear optical characteristics. As a result, a numerical simulation is developed to study the evolution of the soliton inside the non-linear optical fiber with the Fourier Split-Step numerical technique with the real parameters associated with the SQD and the optical fiber.

2. Theoretical Analysis

SQDs systems are zero-dimensional nanostructures that can confine the movement of charge carriers in all spatial dimensions, which present a discrete energy spectrum. The values of the transitions of the dipole moments in this type of systems can cause that the intensity in the interaction between the SQDs and the optical pulse increases significantly in comparison with the atomic systems, which can modify some properties with interesting applications in the area of non-linear optics. At present, different SQDs have been manufactured using different growth techniques and where it has been demonstrated that the optical properties exhibited by this type of structures have a strong influence of their morphology and the materials used in their manufacturing process. [5]. On the other hand, the optical solitons were observed for the first time in media that had a high absorption at specific wavelengths known as resonant optical media, but that at a certain minimum power the medium presented a transparent behavior, for this purpose it is known as self-induced transparency SIT. This effect is experienced by the resonant atoms in a medium, which can be in resonance depending on the frequency of the carrier with respect to the optical transitions given in the material and at the same time some conditions must be satisfied [11]. In the case of SQDs, these generally have extremely large dipole moments, which causes the non-linear interaction between the SQDs and the optical excitation to be greater compared to the atomic systems, so from the point of view of the effects Nonlinear optics, The SIT effect in SQDs systems will be different compared to atomic systems. As an important consequence in the interaction of intense light with SQD systems is the formation of non-linear optical waves such as optical solitons, which acquire certain characteristics in term of the incident light and the morphological characteristics of the SQDs. We will consider a quantum dot as a three-level energy system, where we will denote $|\phi_1\rangle$ as the base state with energy $\varepsilon_{\phi_1}=0$, $|\phi_2\rangle$ is the first excited state or excited state with energy $\varepsilon_{\phi_2}=\varepsilon_x$, where ε_x correspond to the excitonic energy, and $|\phi_3\rangle$ is the second excited state with its corresponding energy value ε_{xx} , which corresponds to the biexcitonic energy.

These states are solutions of the Schrödinger equation $\hat{H}_0|n\rangle=\varepsilon_n|n\rangle$, where $\hat{H}_0=\hbar\nu_o|\phi_2\rangle\langle\phi_2|+\hbar\nu_{xx}|\phi_3\rangle\langle\phi_3|$ [13]. During the excitation of the SQDs, we consider an external light source, composed of a linear optical wave polarized high intensity from a laser and we will study the formation of non-linear optical waves that are re-emitted by the SQDs. Figure 1 shows a schematic of a linearly polarized plane wave incident on SQDs, in the process, due to the interaction of the light with the SQDs, the light is re-emitted in the form of a non-linear wave with special characteristics that will depend on the morphology of the quantum dot and the incident wave.

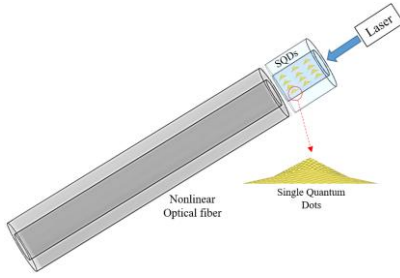


Figure 1. Esquema de la interacción luz – punto cuántico

In the theoretical model, we consider the Hamiltonian corresponding to the interaction of a plane wave linearly polarized with an assembly of quantum dots QDs, of the form: $\hat{H} = \hbar\nu_o |\phi_2\rangle\langle\phi_2| + \hbar\nu_{xx} |\phi_3\rangle\langle\phi_3| - \vec{P} \cdot \vec{E}$. The incident light is considered linearly polarized in the TE mode and has a width T and a frequency $\omega \gg T^{-1}$, with an electric field $\vec{E}(x, y, x, t) = \vec{e}E(x, y, x, t)$ propagating along the z axis, where is a polarization vector in the direction y. In the analytic treatment, it is possible to show that the wave equation for the electric field of the plane wave that affects the assembly of quantum dots and that produces a secondary dipolar field that propagates in it in the same direction as the incident field \vec{E} is given by $-c^2 \frac{\partial^2 E}{\partial z^2} + \eta^2 \frac{\partial^2 E}{\partial t^2} - \frac{\partial^2 E}{\partial x^2} = -4\pi \frac{\partial^2 P}{\partial t^2}$ [13], where $\vec{P}(x, z, t) = N_o \int d\mathcal{G} g(\mathcal{G}_{01}) \langle \vec{\mu} \rangle(\mathcal{G}, x, z, t) + c.c$ y $\langle \vec{\mu} \rangle$ corresponds to the expected value the dipolar moments of the transitions between the levels $|\phi_n\rangle$ y $|\phi_m\rangle$, $g(\nu_{01})$ is a function of distribution of the frequencies of the transitions, which depend on the sizes of the point y $\mathcal{G}_{01} = \mathcal{G}_0 - \mathcal{G}_1$ is the detuning, η is the refractive index of the semiconductor, and y N_o is the density of quantum dots. There are alternative methods such as the Bloch Vector and the optical Bloch equations that relate the optical phenomena that occur in this type of transitional problems with the density matrix.

In concordance, we can use the Liouville equation to determine the elements of the density matrix[14].

$$i\hbar \frac{\partial \rho}{\partial t} = [H, \rho] = \sum_l (\langle n | H | l \rangle \rho_{lm} - \rho_{nl} \langle l | H | m \rangle); \quad n, m, l = 1, 2, 3 \quad (1)$$

Substituting the perturbed Hamiltonian in equation 1 and taking into account that $\langle n | \hat{V} | m \rangle = V_{nm} = -\mu_{nm} \cdot E$ we obtain a system of equations for the elements of the density matrix.

On the other hand, the electric field of the pulse is considered in the form $E = \sum_{l=\pm 1} \hat{E}_l \varphi_l$, where

$\varphi_l = \exp[il(kz - \mathcal{G}_l t)]$ and \hat{E}_l it is the complex amplitude of slow envelope of the electric field.

Substituting the equation for the electric field and to guarantee E real, we choose $\hat{E}_l = \hat{E}_{-l}^*$, which is widely used in the study of nonlinear waves. Using the slow envelope approach, which relies on the envelopes \hat{E}_l of the pulses vary smooth enough in space and time, that is,

$$\left| \frac{\partial \hat{E}_l}{\partial t} \right| \ll \nu_l |\hat{E}_l|, \quad \left| \frac{\partial \hat{E}_l}{\partial z} \right| \ll k |\hat{E}_l|, \quad \text{then we can eliminate the second derivatives of the amplitudes in the}$$

wave equation, taking this form:

$$\sum_{l=\pm 1} \varphi_l \left[l^2 \left((ck)^2 - \eta^2 \mathcal{G}_1^2 \right) \hat{E}_l - 2iklc^2 \frac{\partial \hat{E}_l}{\partial z} - 2il\eta^2 \mathcal{G}_1 \frac{\partial \hat{E}_l}{\partial t} \right] = -4\pi l^2 \mathcal{G}_1^2 N_o \left[(\mu_{12} \hat{\rho}_{21} + \mu_{23} \hat{\rho}_{32}) e^{i(kx - \mathcal{G}_1 t)} \right] + c.c \quad (2)$$

The elements ρ_{11} , ρ_{22} y ρ_{33} give rise to the populations in the states $|\phi_1\rangle, |\phi_2\rangle$ y $|\phi_3\rangle$ correspondingly, and non-diagonal elements ρ_{nm} whit $n \neq m$, they contain the relative phase between the states that describe the atomic coherence, so they can be written in terms of φ . The system of equations (1) would be of the next form, where we have introduced $A(z) = \frac{\mu_{12}}{\hbar} \int_{-\infty}^t E(z,t) dt$, that corresponds to the area of the nonlinear optical pulse and $\delta = \mu_{23} / \mu_{12}$:

$$\begin{aligned} i\hbar\dot{\rho}_{11} &= \hat{A}(\hat{\rho}_{21} - \hat{\rho}_{12}); & i\hbar\dot{\rho}_{21} &= \hat{A}(\hat{\rho}_{11} + \hat{\rho}_{22} - \delta\hat{\rho}_{31}); & i\hbar\dot{\rho}_{22} &= \hat{A}(\hat{\rho}_{21} - \hat{\rho}_{12} - \delta\hat{\rho}_{32} + \delta\hat{\rho}_{23}); \\ i\hbar\dot{\rho}_{33} &= \hat{A}(\hat{\rho}_{32} - \hat{\rho}_{23}); & i\hbar\dot{\rho}_{32} &= \hat{A}(-\delta\hat{\rho}_{22} + \delta\hat{\rho}_{33} + \hat{\rho}_{31}); \end{aligned} \quad (3)$$

The solution of the system of equations (5), allows the calculation of the elements of the right side of equation (2). As the diagonal elements of the matrix give rise to the populations in the states, under the condition of the system in the base state, $\rho_{11}=1$, $\rho_{22}=0$ y $\rho_{33}=0$, the equations (3) take the form

$$\hat{\rho}_{21} = \frac{i}{2d^3} (\sin 2dA + 2\delta^2 \sin dA) \text{ and } \hat{\rho}_{32} = \frac{i\delta}{2d^3} (\sin 2bA - 2\sin dA), \text{ where } d = (1 + \delta^2)^{\frac{1}{2}}.$$

So we get the well-known double-Sine-Gordon equation when $\delta=0$, we obtain $\psi_{xx} + \frac{\eta}{c}\psi_{xx} + \frac{\pi g \mu_{12}^2 N_0}{\hbar \eta^2} \sin(2\psi) = 0$. Is possible to solve this equation by using a transformation of coordinates for the time of the form $\xi = t - z/v$, where v is the velocity of the pulse. The approach is to assume that there is a solution to determine the complex amplitude of the non-linear Schrödinger NLS equation that has the form $\chi(z,t) = U_1 S(\xi) e^{i(kz - \theta t)}$ and developing this transformation and using the approximation $\chi(z,t)$ we obtain an ordinary differential equation

$$\frac{d^2 \chi}{d\xi^2} = 2\pi g \mu_{12}^2 N_0 \left[\hbar \eta^2 \left(\frac{c - \eta v}{\eta v} \right) \right]^{-1} \sin \chi \quad (4)$$

In real quantum dots the delta of frequencies between levels and between these and the pulse are different from zero, therefore it is assumed that the transition from level 1 to 2 and from level 2 to level 3 are very close to each other and the pulse frequency. Taking into account the approximation of a non-resonant excitation with a constant detuning, i.e. $\mathcal{G}_{soliton} - \mathcal{G}_0 - \mathcal{G}_1 \approx \mathcal{G}_0 - \mathcal{G}_1 = \mathcal{G}$ y $\mu_{12} = \mu_{23}$, then

solution of equation (9) is known as *solitonic solution* $\chi(z,t) = \frac{2\hbar}{\mu_{12} T} \operatorname{sech} \left(\frac{\xi}{T} \right)$, whit

$$T = \left(\frac{\hbar \eta^2}{2\pi g \mu_{21}^2 N_0} \left(\frac{c}{\eta v - 1} \right) \right)^{\frac{1}{2}} \text{ that corresponds to the width of the pulse reemitted by the SQD [15].}$$

3. Resultados

The theoretical results of equation solitonic that was obtained from the double-sine-Gordon equation, show that the excitation of SQDs from an intense non-linear wave can produce optical solitons as a consequence of the interaction of light with the SQDs. During the non-linear interaction process, the SQDs are considered as a three-level energy quantum system, in which the optical transitions are given from the ground state to the excitonic and biexcitonic states. The allowed transitions between the ground state and the excitonic and biexcitonic states have a much lower dipole moment than the transition between the ground state and the background of the exciton band. On the other hand, the

characteristics of the light reemitted by the assembly of quantum dots in the form of optical solitons will depend on the intensity of the incident light, whose minimum value to form the optical solitons could be determined specifically depending on the nature of the SQDs. In this way, the soliton reemitted will depend on the refractive index of the semiconductor η , the dipole moment corresponding to the transitions between the base state and the excitonic state or between the excitonic state and the biexcitonic state μ_{12} , taking into account that we have considered an energy system of three equidistant levels. On the other hand, there is also a dependence on the external excitation frequency f and the density of quantum dots N_0 . On the other hand, for the study of the propagation of short optical pulses through non-linear optical fibers, the non-linear Schrodinger NLSE equation is used, which takes into account the effects of the length of the fiber, the dispersion effect of group speed and non-linear optical effects as a consequence of the high intensity of light. The Fourier Split-Step method is a pseudo-spectral technique that is extremely useful due to its rapid and good accuracy in calculations. In general, this method obtains an approximate solution of the propagation equation, assuming that dispersion effects and non-linear effects act independently along the fiber in very small steps [16]. This technique was used to simulate the evolution of solitaires that are re-emitted by the SQD. In the proposal that we propose in this research, we assume that the assembly of quantum dots is coupled to the optical fiber, which allows the light to re-emit by the QDs as a consequence of non-linear interaction between the light and the SQDs is directly coupled to the sea the core of the fiber and in this way the light will be guided by total internal reflection on the interior of the fiber, which can be clearly seen in figure 1. For the simulation, we have considered. In Figure 2, the profile of the soliton re-emitted by the SQDs system for different concentrations of quantum dots is observed. In the results, it is observed that the peak intensity decays at higher density values. An explanation to this effect can realize that it can increase the QDs per unit of volume, it can increase the effects of the absorption of the SQD, re-emitting with a lower intensity. On the other hand, we must bear in mind that the mathematical modeling proposed requires that the density N_0 of quantum dots is lower, so that the interactions of QDs in the Hamiltonian are omitted. For the simulation we have proposed SQDs with pyramidal morphology of InAs / GaAs manufactured with a cylindrical symmetry with the parameters: $\eta = 3.3$, $\mu_{12} = 1.9 \times 10^{-28} \text{ Cm}$, $\lambda = 850 \text{ nm}$, $v = 1.7 \times 10^8 \text{ m/s}$.

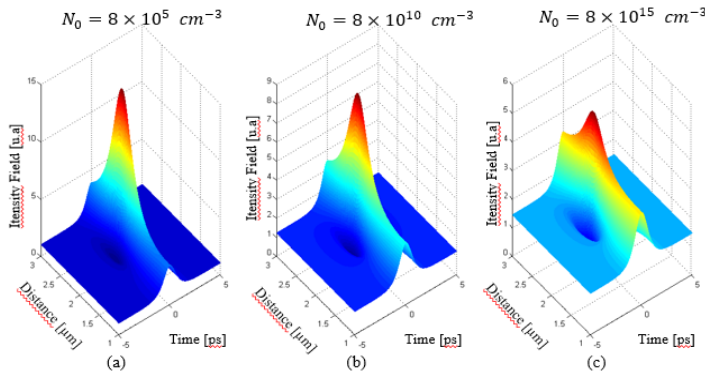


Figure 2: Simulation of the soliton generated by the SQDs system with different densities of QDs. a) $8 \times 10^5 \text{ cm}^{-3}$. B) $8 \times 10^{10} \text{ cm}^{-3}$. C) $8 \times 10^{15} \text{ cm}^{-3}$

For the simulation of the soliton evolution in the optical fiber we have used the split-step Fourier method on a non-linear standard optical fiber with the parameters: fiber attenuation $\alpha = 0.1 \text{ dB/km}$, non-linear fiber parameter $\gamma = 0.3 \text{ W/m}$ and second order dispersion $\beta_2 = -20 \times 10^{-27} \text{ s}^2/\text{m}$. From the simulation results, figure 3a shows the input pulse in the fiber while in figure 3b the pulse is observed at a distance traveled of 1000 m.

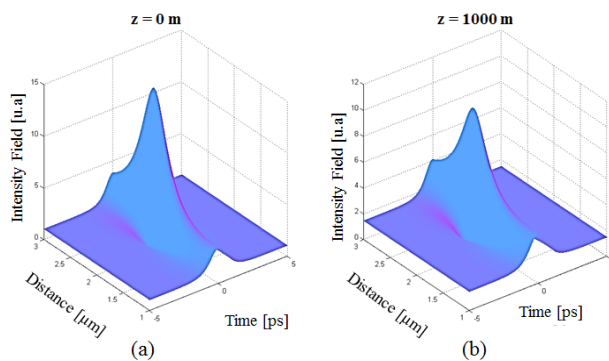


Figure 3:
Evolution of
soliton through
non-linear optical
fiber

4. Conclusiones

Through the present investigation, it can be concluded that it is possible to produce solitonic pulses in quantum dots embedded in nonlinear optical fibers with the purpose of propagating fields without losses over long distances. The analytical treatment formulated in this work, would guarantee the generation of solitonic optical pulses, which would be modulated according to the amplitude of the pulse and its morphology. Likewise, it is proposed the manufacture of this type of nanostructures with different morphologies inserted in optical fibers that allow the propagation of light without losses over long distances

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