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## **A spatio-temporal statistical downscaling approach to deriving extreme rainfall IDF relations at ungauged sites in the context of climate change**

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**Abstract.** This paper proposes an efficient spatio-temporal statistical downscaling approach for estimating IDF relations at an ungauged site using daily rainfalls downscaled from global climate model (GCM) outputs. More specifically, the proposed approach involves two steps: (1) a spatial downscaling using scaling factors to transfer the daily downscaled GCM extreme rainfall projections at a regional scale to a given ungauged site and (2) a temporal downscaling using the scale-invariance GEV model to derive the distribution of sub-daily extreme rainfalls from downscaled daily rainfalls at the same location. The feasibility and accuracy of the proposed approach were evaluated based on the climate simulation outputs from 21 GCMs that have been downscaled by NASA to a regional 25-km scale for two different RCP 4.5 and 8.5 scenarios and the observed extreme rainfall data available from a network of 15 raingauges located in Ontario, Canada. The jackknife technique was used to represent the ungauged site conditions. Results based on different statistical criteria have indicated the feasibility and accuracy of the proposed approach.

**Keywords:** climate change impacts, extreme rainfalls, GEV distribution, IDF curves, scale invariance, ungauged sites

### **1 Introduction**

Extreme rainfall intensity-duration-frequency (IDF) relations are an essential tool for the design of various hydraulic structures, especially, for urban drainage systems [1, 2]. The derivation of extreme rainfall IDF relations at a location of interest where data is unavailable (i.e., an ungauged site) is one of the most challenged tasks in current engineering practices, particularly in the context of climate change [3-5]. This IDF estimation requires hence a new spatial rainfall modeling approach to establishing an accurate linkage between daily climate projections from global (or regional) climate models and daily extreme rainfall (ER) processes at an ungauged site. Furthermore, since this linkage could only be established at the daily timestep due to the limitations in detailed physical modeling and computational capability of current climate models, hence it requires an additional temporal rainfall modeling approach to simulating ER processes over a wide range of time scales (e.g., one day to several minutes) for the derivation of the sub-daily ERs from the downscaled daily values.

To downscale the GCM coarse grid climate projections to much finer spatial resolutions at regional (or local) scales for reliable assessments of climate change impacts, different downscaling methods have been proposed to resolve this scale discrepancy [6-10]. These methods are generally classified into two broad categories: dynamical downscaling (DD) and statistical downscaling (SD). It has been widely recognized that the SD methods offer several practical advantages over the DD procedures, especially in terms of flexible adaptation to specific study purposes, and inexpensive computing

resource requirement [11, 12]. Depending on the scales of interest for climate change and hydrologic process impacts and adaptation studies, SD methods can be used to spatially disaggregate GCM projected outputs to regional scales (multiple gridded) or local/point scales (a single site or multi-sites) [6, 9, 13]. When dealing with a large ensemble of GCMs, the gridded SD methods are often in favor because of their computational efficiency and effectiveness in producing physically plausible hydro-climatology data [13, 14]. Additionally, for regions without observed data, this SD approach is a promising tool as it could be used to re-construct historical as well as projected ERs for these ungauged locations. However, before it could be used to assess impacts of climate change on daily ERs at a given ungauged site, it is critical to correct the bias associated with the downscaled data to the location of interest.

To derive the statistical properties and distributions of the sub-daily ER series from those of the daily values at a gauged site, the scale-invariance (or scaling) method has been widely applied [15-22]. Scale invariance implies that the statistical properties of ERs over different time scales are related to each other by an operator involving only the scale ratio and the scaling exponent [23-26]. A novel scaling probability weighted moments-based GEV model has been recently proposed by [27] and has been shown to outperform other existing scaling models. For ungauged site, there is no procedure to deriving the distributions of projected sub-daily ERs from daily values using the scale-invariance approach reported in the scientific literature yet.

In view of the above issues, the present paper proposes an efficient spatio-temporal statistical downscaling approach for estimating IDF relations at an ungauged site in the context of climate change. The approach uses the daily downscaled CMIP5 climate projections available at the regional scale (approximately 25 x 25 km) that has been produced by NASA based on the outputs from the 21 global climate models (GCMs) using the bias-correction spatial disaggregation technique [28]. The NASA daily extreme rainfalls are first spatially downscaled to a given ungauged site. Then the scale-invariance GEV model is utilized to downscale the daily to sub-daily ERs at the same location. This spatio-temporal statistical downscaling procedure is described in section 2. The feasibility and accuracy of the proposed approach are evaluated in section 3 using the IDF data from a network of 15 raingauges with long rainfall records and from 69 neighbouring stations located in Ontario (Canada). The jackknife technique was used to represent the ungauged site condition for these 15 study sites. A summary of research findings is provided in section 4.

## 2 A spatio-temporal statistical downscaling (STSD) approach to estimating IDF relations at ungauged sites

### 2.1 Spatial statistical downscaling of daily extreme rainfalls at an ungauged site

To construct the IDF relations at an ungauged location, the first step is to derive the distribution of daily ER series at that location based on the available daily ER series at the regional scale. In general, a spatial statistical downscaling technique is based on the perspective that local information could be derived by determining a statistical model which relates regional climate variables to any local weather variables. In the present study, the scaling factor is used for transferring the NASA extreme rainfall at the regional 25-km scale,  $\hat{X}$ , to the ER at a given ungauged site,  $X_i$  [29]. The regional value is adjusted by a scaling factor as defined by:

$$X_i(F) = \eta_i \cdot \hat{X}(F); \quad (1)$$

where  $F$  is the cumulative probability of interest and  $\eta_i = \mu_i/\hat{\mu}$  is the scaling factor at site  $i$ ;  $\mu_i$  and  $\hat{\mu}$  are the expected means (i.e. estimated based on the sample means) of the daily ER series at the ungauged site of interest  $i$  and at the grid containing that site, respectively. Note that  $\mu_i$  must be interpolated from sites where daily ER records are available. In the present study, for the sake of simplicity, the popular inverse distance weighted (IDW) method was applied for the interpolation.

## 2.2 Temporal statistical downscaling of sub-daily extreme rainfalls at an ungauged site

After obtaining the daily extreme rainfall series at the location of interest, the second step is to derive the statistical properties of sub-daily ER series at the same site. To do this, the scale-invariance GEV/PWM model [27] was used as this distribution has been commonly used for describing the distribution of ERs and for constructing the IDF relations [30]. The quantile,  $X_T$ , corresponding to certain return periods,  $T = 1/(1 - F)$ , can be obtained using the following expression:

$$X_T = \xi + \frac{\alpha}{\kappa} \{1 - [-\ln(F)]^\kappa\} \quad (2)$$

in which  $\xi$ ,  $\alpha$ , and  $\kappa$  are the location, scale, and shape parameters respectively; and  $F$  is the cumulative probability of interest.

The probability weighted moments (PWMs) and its linear combination L-moments were used for estimation of the GEV parameters [31]. The general  $r^{\text{th}}$  order of PWM,  $B_r$ , of the GEV distribution can be expressed as in Eqn.(3) [32]. For a simple scaling process, it can be shown that the scale-invariance property of the PWMs over different rainfall durations can be expressed as shown in Eqn.(4). This infers that scaling exponents are constant across all PWM orders for the same scaling regime [18].

$$B_r = M_{1,r,0} = E[X \{F(X)\}^r] = (r + 1)^{-1} \left( \xi + \frac{\alpha}{\kappa} \{1 - (r + 1)^{-\kappa} \Gamma(1 + \kappa)\} \right) \quad (3)$$

$$B_r(\lambda t) = \lambda^{\beta r} B_r(t) = \lambda^\beta B_r(t) \quad (4)$$

in which  $\Gamma(\cdot)$  is the gamma function;  $\beta_r = \beta$ , and  $\beta = \beta_0^{PWM}$  is the scaling exponent or PWM order  $r = 0$  (i.e. the mean).

Furthermore, for a simple scaling process, it can be shown that the statistical properties of the GEV distribution for two different time scales  $t$  and  $\lambda t$  are related as follows:

$$\kappa(\lambda t) = \kappa(t); \alpha(\lambda t) = \lambda^\beta \alpha(t); \xi(\lambda t) = \lambda^\beta \xi(t); X_T(\lambda t) = \lambda^\beta X_T(t); \quad (5)$$

Hence, based on these relationships it is possible to derive the short-duration ERs data and to construct the IDF relations for a local ungauged site. Note that similar to the mean, the scaling exponent at the ungauged site must be interpolated from sites where daily and sub-daily ER records are available. Again, the well-known IDW method was applied for this interpolation.

## 3 Numerical application

### 3.1 Study sites and data

The climate simulation outputs from 21 global climate models (GCMs) conducted under the Coupled Model Inter-comparison Project Phase 5 (CMIP5) and the observed IDF data in the period of 1961-2005 from a network of 15 raingauges located in the Ontario province, Canada, were selected for this study (see Figure 1). The climate simulation outputs have been statistically downscaled by NASA (i.e., NASA Earth Exchange) from the global scales (a few degrees or  $10^2$  km) to the regional scale (approximately 25 km x 25 km) for two different Representative Concentration Pathways scenarios (i.e. RCP 4.5 and 8.5) based on the bias-correction spatial disaggregation approach [28]. Observed IDF data at each site consists of annual maximum rainfall series for nine different durations (ranging from 5 minutes to 1440 minutes). Note that the observed IDF data have been provided by the Environment Canada to produce the at-site IDF relations for the various practical engineering application purposes. The jackknife technique was used to represent the ungauged site condition at the study sites. In addition to these sites, IDF data from other 69 neighbouring stations were also used for the interpolation of the means and scaling exponents at the ungauged sites. Data of 1961-1990 were used for the calibration of the scaling factors while those of 1991-2005 were used for the validation of the calibrated scaling factors. Selection of these stations relied on the quality of the data,

the adequate length of available historical ER records, and the representative spatial distribution of raingages.

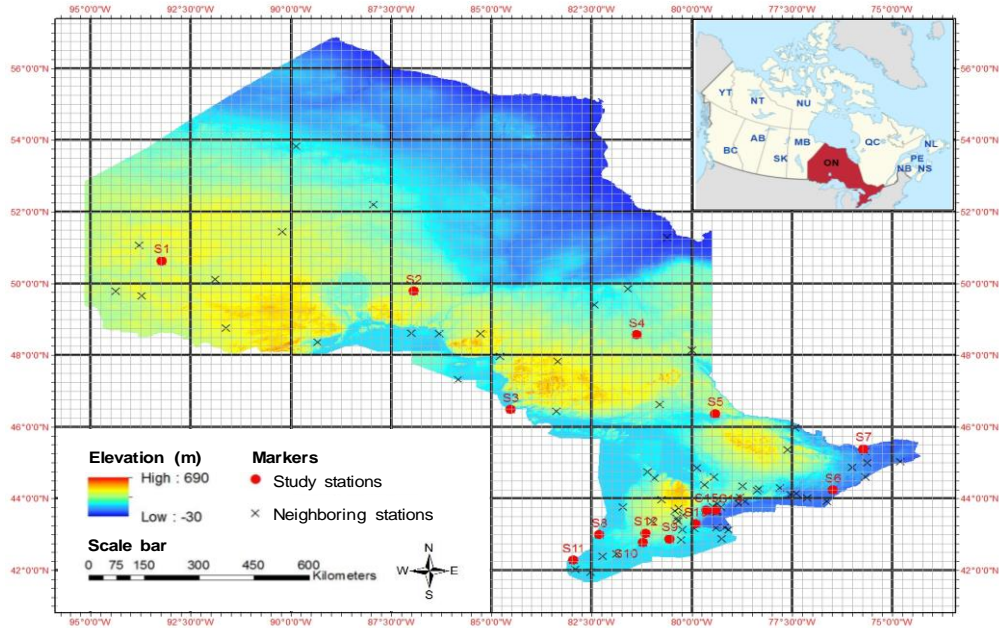


Figure 1. Locations of the 15 study raingages (red circle markers) and 69 neighboring stations (black cross markers) used for the study. The bold black lines show a common GCM grid of 2.5°x2°, while the gray lines show the NASA grid of 0.25°x0.25°. The provincial digital elevation model was obtained from [33]

### 3.2 Assessment criteria

To evaluate the feasibility and accuracy of the proposed STSD approach to estimating short-duration extreme design rainfalls (or IDF relations) at an ungauged site in the context of climate change, different graphical visualization and goodness-of-fit (GOF) tests were used. Q-Q plot was used for the graphical comparisons while three common GOF indices were selected for the numerical comparisons. These criteria include the root mean square relative error (RMSEr), the mean absolute relative deviation (MADr), and the correlation coefficient (CC) as follows:

$$RMSEr = \left[ \frac{1}{n} \sum \left\{ \frac{(x_i - y_i)}{x_i} \right\}^2 \right]^{1/2} \quad (6)$$

$$MADr = \frac{1}{n} \sum \left\{ \frac{|x_i - y_i|}{x_i} \right\} \quad (7)$$

$$CC = \frac{\sum \{(x_i - \bar{x})(y_i - \bar{y})\}}{\{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2\}^{1/2}} \quad (8)$$

where  $x_i, i = 1, 2, \dots, n$  are the observed values and  $y_i, i = 1, 2, \dots, n$  are the estimated values for the same probability level  $p_i$ ;  $n$  is the sample length;  $\bar{x}$  and  $\bar{y}$  denote the average value of the observed and estimated quantiles, respectively.

### 3.3 Results and discussion

#### 3.3.1 Derivation of daily extreme rainfalls at an ungauged site

To transfer the NASA extreme rainfalls at the 25-km regional scale,  $\hat{X}$ , to a given ungauged site, a scaling factor was used as indicated by Eq. (1). Figure 2 shows the comparisons of different GOF test results between the estimated (i.e., regional and the bias-corrected values) and the observed ERs for both the calibration (1961-1990) and the validation periods (1991-2005). It is important to note that there is a systematic bias between the ERs at regional scales and at a local site. Indeed, the correlation

coefficients between the regional and observed values are high (about 30%) but the errors are also large (higher than 0.9) for both the calibration and validation periods. The use of a transfer function (i.e. bias correction) is thus necessary. Furthermore, it can be clearly seen that the bias-corrected ERs derived for an ungauged site using the estimated scaling factor produced lower values of RMSEr and MADr as well as higher values of CC as compared to the raw data (i.e. 25x25 km regional values) obtained directly from NASA. In addition, the low values of RMSEr and MADr (about 10% and 15% or less for the calibration and validation respectively) and high values of CC (about 0.95 or higher) have indicated the feasibility and accuracy of the proposed spatial downscaling (or bias correction) approach in the estimation of extreme design rainfalls for an ungauged location.

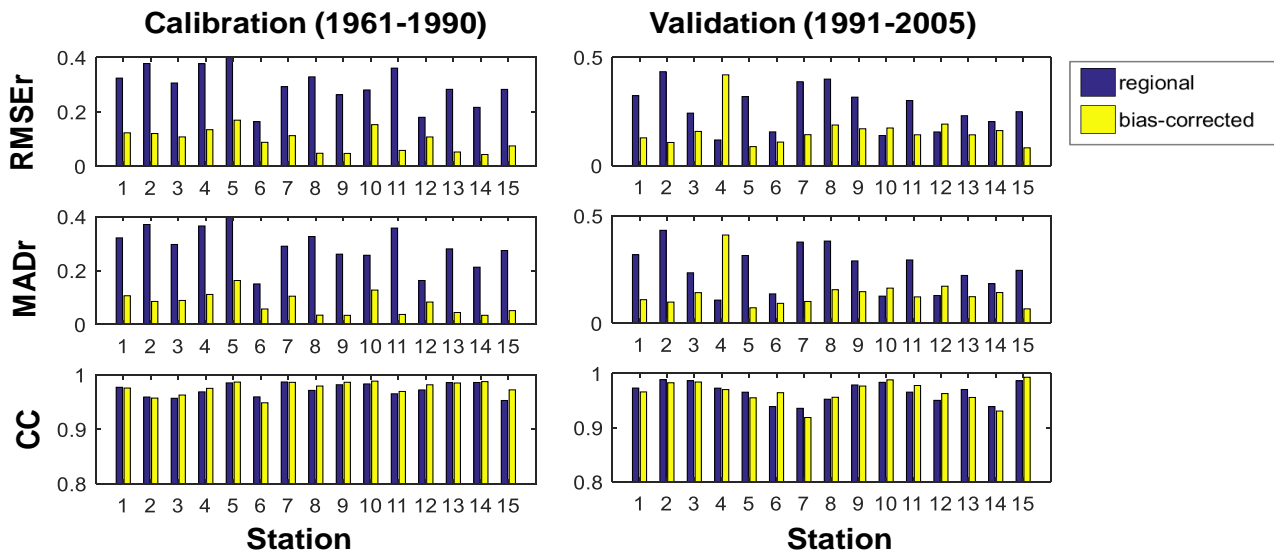


Figure 2. Comparisons of GOF results between observed and estimated (i.e., regional and bias-corrected) ERs at the 15 study sites for the calibration (1961-1990) and validation (1991-2005) periods.

### 3.3.2 Derivation of sub-daily extreme rainfalls at an ungauged site

To obtain the sub-daily extreme rainfall series from the daily ER series at a given site, the proposed scale-invariance GEV/PWM method was applied. Different graphical visualization and goodness-of-fit (GOF) tests were used to evaluate the feasibility and accuracy of this method. For purpose of illustration, Figure 3 shows the probability plots of the computed design extreme rainfalls  $X_T$  (mm) for two different durations at station #13 – the Hamilton RBG CS station for both the calibration (1961-1990) and validation (1991-2005) periods respectively. Uncertainty associated with the estimation of the extreme design rainfalls is displayed in the form of standard boxplots. It can be seen that the distributions of the estimated sub-daily ERs derived based on the distribution of daily ERs and the scale-invariance GEV/PWM model agreed well with the observed data. Figure 4 shows the Q-Q plots of the estimated extreme design rainfalls derived based on the scale-invariance GEV/PWM and the at-site frequency analysis using the GEV distribution for different rainfall durations and return periods for all 15 selected stations. Note that the median values of the results from 21 GCMs were used for the computation. A numerical comparison was conducted to evaluate the results using the three selected GOF indices (i.e. RMSEr, MADr, and CC) for all sites as shown in Table 1. The low values of RMSEr and MADr as well as the high values of CC have indicated the feasibility and accuracy of the proposed temporal GEV/PWM statistical downscaling in the estimation of the extreme design rainfalls for a given ungauged location. Note that, for accuracy, only the estimated quantiles within the twice sample lengths (i.e. upto 50-year and 25-year return periods for the calibration and validation respectively) were used for comparisons .

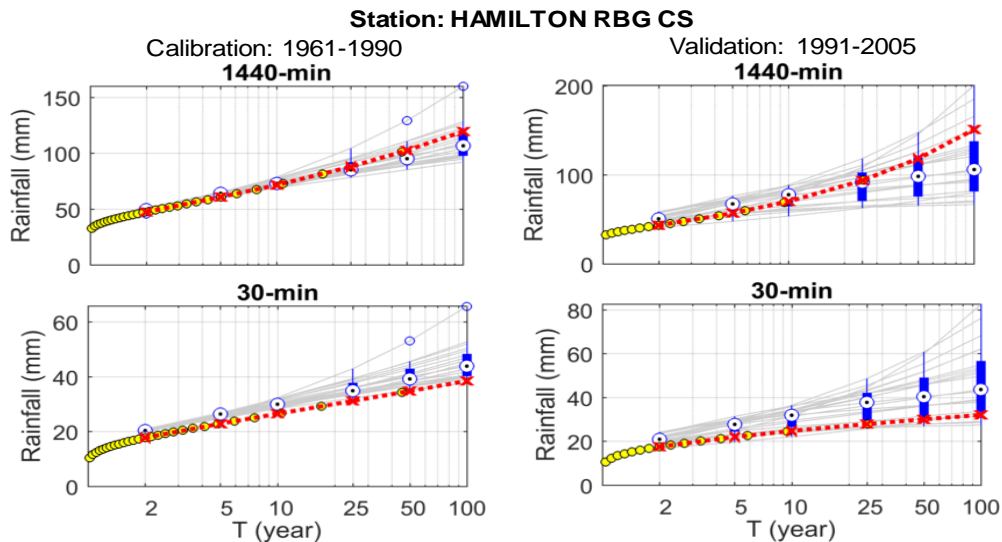


Figure 3. Probability plots between the observed and estimated short-duration ERs at station S13-Hamilton RBG CS. Yellow circle markers show the empirical probability plots while red cross markers and dash lines show the at-site frequency analysis plots. The gray lines and boxplots show the derived distributions of short-duration ERs based on the proposed STSD approach.

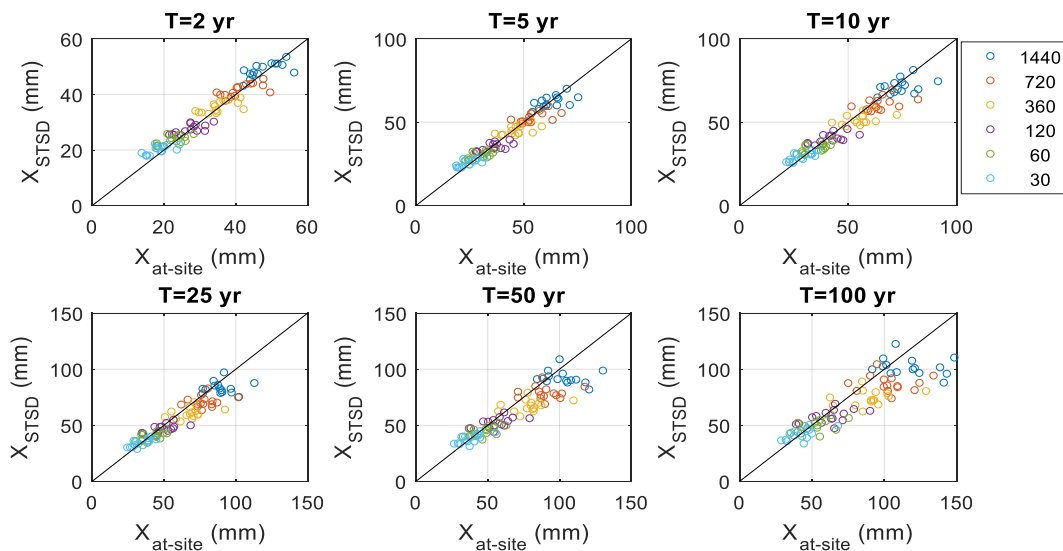


Figure 4. Q-Q plots of the estimated ERs based on the proposed STSD procedure ( $X_{STSD}$ , mm) and the at-site frequency analysis ( $X_{at-site}$ , mm) using the GEV distribution for different rainfall durations (from  $D=30$  to 1440 minutes) and for different return periods ( $T=2$  to 100 years) for the 1961-1990 calibration period.

#### 4 Conclusions

The major findings of the present study can be summarized as follows:

- a) An innovative spatio-temporal statistical downscaling (STSD) procedure for estimating short-duration extreme design rainfalls (or IDF relations) at an ungauged site in the context of climate change was proposed. The proposed STSD approach involves two steps: (1) a spatial downscaling method based on a bias-correction scaling factor to transfer the daily downscaled GCM extreme rainfall projections at a regional scale to a given ungauged site; and (2) a temporal downscaling procedure using the scale-invariance GEV/PWM model to derive the distribution of sub-daily from daily extreme rainfalls at the same location.

b) A numerical application of the STSD method was carried out using IDF data from 15 raingauge stations and 69 neighboring stations located in Ontario (Canada). The jackknife technique was used to represent the ungauged site condition at the 15 study raingauges. Different graphical displays (i.e., probability plots, Q-Q plots, and boxplots) and three dimensionless goodness-of-fit (GOF) indices (i.e., RMSEr, MADr, and CC) were utilized to assess the performance of the model. In general, the estimated values by the proposed STSD agree very well with the observed data. More specifically, the low values of RMSEr and MADr and high values of CC have indicated the accuracy of the proposed STSD in the estimation of short-duration extreme design rainfalls for an ungauged location in the context of a changing climate.

The inferences made in this paper are based upon a case study using the IDF data from Ontario and NASA daily downscaled climate projections available at the regional 25-km scale. Similar studies should be carried out to assess the feasibility and accuracy of the proposed approach to extreme design rainfall estimation in other climatic regions.

Table 1 Goodness-of-fit test results for both calibration and validation periods

T (year)	Calibration period 1961-1990					Validation period 1991-2005			
	2	5	10	25	50	2	5	10	25
RMSEr (%)	10.3	10.3	11.4	13.9	16.3	15.9	15.2	16.5	20.2
MADr (%)	8.0	8.1	9.3	11.8	13.8	13.4	12.5	12.8	15.9
CC (dmnl)	0.965	0.958	0.950	0.931	0.910	0.929	0.903	0.885	0.866

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### References

- [1] WMO, Guide to hydrological practices, volume II: Management of water resources and application of hydrological practices, 6th edition, World Meteorological Organization, Geneva, Switzerland, 2009.
- [2] D.M. Hershfield, Rainfall frequency atlas of the United States for durations from 30 minutes to 24-hours and return periods from 1 to 100 years, Technical Paper 40 U.S. Weather Bureau, Washington, D.C., 1961.
- [3] P. Willems, J. Olsson, K. Arnbjerg-Nielsen, S. Beecham, A. Pathirana, I.B. Gregersen, H. Madsen, V.-T.-V. Nguyen, Impacts of climate change on rainfall extremes and urban drainage systems, IWA Publishing, London, UK, 2012.
- [4] H. Madsen, D. Lawrence, M. Lang, M. Martinkova, T.R. Kjeldsen, Review of trend analysis and climate change projections of extreme precipitation and floods in Europe, *J. Hydrol.*, 519 (2014) 3634-3650.
- [5] S.P. Simonovic, A. Schardong, D. Sandink, R. Srivastav, A web-based tool for the development of Intensity Duration Frequency curves under changing climate, *Environmental Modelling & Software*, 81 (2016) 136-153.
- [6] R.L. Wilby, C.W. Dawson, E.M. Barrow, SDSM - A decision support tool for the assessment of regional climate change impacts, *Environ. Model. Softw.*, 17 (2002) 147-159.
- [7] H.J. Fowler, S. Blenkinsop, C. Tebaldi, Linking climate change modelling to impacts studies: recent advances in downscaling techniques for hydrological modelling, *Int. J. Climatol.*, 27 (2007) 1547-1578.
- [8] D. Maraun, F. Wetterhall, A.M. Ireson, R.E. Chandler, E.J. Kendon, M. Widmann, S. Brienen, H.W. Rust, T. Sauter, M. Themeßl, V.K.C. Venema, K.P. Chun, C.M. Goodess, R.G. Jones, C. Onof, M. Vrac, I. Thiele-Eich, Precipitation downscaling under climate change: Recent developments to bridge the gap between dynamical models and the end user, *Rev. Geophys.*, 48 (2010) n/a-n/a.
- [9] M. Khalili, V.T. Van Nguyen, An efficient statistical approach to multi-site downscaling of daily precipitation series in the context of climate change, *Climate Dynamics*, (2016).



- [10] E. Gooré Bi, P. Gachon, M. Vrac, F. Monette, Which downscaled rainfall data for climate change impact studies in urban areas? Review of current approaches and trends, *Theor. Appl. Climatol.*, 127 (2017) 685-699.
- [11] C.Y. Xu, From GCMs to river flow: A review of downscaling methods and hydrologic modelling approaches, *Prog. Phys. Geogr.*, 23 (1999) 229-249.
- [12] C. Prudhomme, S.C. Nick Reynard, Downscaling of global climate models for flood frequency analysis: Where are we now?, *Hydrol. Processes*, 16 (2002) 1137-1150.
- [13] A.T. Werner, A.J. Cannon, Hydrologic extremes -- an intercomparison of multiple gridded statistical downscaling methods, *Hydrol. Earth Syst. Sci.*, 20 (2016) 1483-1508.
- [14] A.W. Wood, L.R. Leung, V. Sridhar, D.P. Lettenmaier, Hydrologic implications of dynamical and statistical approaches to downscaling climate model outputs, *CLIM. CHANGE*, 62 (2004) 189-216.
- [15] P. Burlando, R. Rosso, Scaling and multiscaling models of depth-duration-frequency curves for storm precipitation, *J. Hydrol.*, 187 (1996) 45-64.
- [16] V.T. Nguyen, T.D. Nguyen, F. Ashkar, Regional frequency analysis of extreme rainfalls, *Water science and technology : a journal of the International Association on Water Pollution Research*, 45 (2002) 75-81.
- [17] V.T.V. Nguyen, T.D. Nguyen, A. Cung, A statistical approach to downscaling of sub-daily extreme rainfall processes for climate-related impact studies in urban areas, in: H.H.P. Fang, J.H.W. Lee (Eds.) *Water Science and Technology: Water Supply*, 2007, pp. 183-192.
- [18] P.-S. Yu, T.-C. Yang, C.-S. Lin, Regional rainfall intensity formulas based on scaling property of rainfall, *J. Hydrol.*, 295 (2004) 108-123.
- [19] J. Bougadis, K. Adamowski, Scaling model of a rainfall intensity-duration-frequency relationship, *Hydrol. Processes*, 20 (2006) 3747-3757.
- [20] J. Blanchet, D. Ceresetti, G. Molinié, J.D. Creutin, A regional GEV scale-invariant framework for Intensity–Duration–Frequency analysis, *J. Hydrol.*, 540 (2016) 82-95.
- [21] H. Ghanmi, Z. Bargaoui, C. Mallet, Estimation of intensity-duration-frequency relationships according to the property of scale invariance and regionalization analysis in a Mediterranean coastal area, *J. Hydrol.*, 541 (2016) 38-49.
- [22] M.T. Vu, V.S. Raghavan, S.Y. Liong, Deriving short-duration rainfall IDF curves from a regional climate model, *Natural Hazards*, (2016).
- [23] V.K. Gupta, E. Waymire, Multiscaling properties of spatial rainfall and river flow distributions, *Journal of Geophysical Research: Atmospheres*, 95 (1990) 1999-2009.
- [24] D. Veneziano, C. Lepore, The scaling of temporal rainfall, *Water Resour. Res.*, 48 (2012) n/a-n/a.
- [25] D. Schertzer, I. Tchiguirinskaia, S. Lovejoy, P. Hubert, No monsters, no miracles: in nonlinear sciences hydrology is not an outlier!, *Hydrol. Sci. J.*, 55 (2010) 965-979.
- [26] S. Lovejoy, D. Schertzer, *The Weather and Climate: Emergent Laws and Multifractal Cascades*, Cambridge University Press, Cambridge, 2012.
- [27] T.-H. Nguyen, V.-T.-V. Nguyen, A novel scale-invariance generalized extreme value model based on probability weighted moments (GEV/PWM) for estimating extreme design rainfalls in the context of climate change, *World Environmental and Water Resources Congress 2018Minneapolis, Minnesota, US, 2018*.
- [28] B. Thrasher, E.P. Maurer, C. McKellar, P.B. Duffy, Technical Note: Bias correcting climate model simulated daily temperature extremes with quantile mapping, *Hydrol. Earth Syst. Sci.*, 16 (2012) 3309-3314.
- [29] T.-H. Nguyen, V.-T.-V. Nguyen, Assessment of climate change impacts on extreme rainfall intensity-duration-frequency (IDF) relations using downscaled CMIP5 climate projections, *World Environmental and Water Resources Congress 2018Minneapolis, Minnesota, US, 2018*.
- [30] M. Schaefer, Regional analyses of precipitation annual maxima in Washington State, *Water Resour. Res.*, 26 (1990) 119–131.
- [31] J.R.M. Hosking, J.R. Wallis, *Regional Frequency Analysis: An Approach Based on L-Moments*, Cambridge University Press, Cambridge, UK, 1997.
- [32] J.R.M. Hosking, J.R. Wallis, E.F. Wood, Estimation of the Generalized Extreme-Value Distribution by the Method of Probability-Weighted Moments, *Technometrics*, 27 (1985) 251-261.
- [33] LIO, Land Information Ontario. <https://www.ontario.ca/page/land-information-ontario>, 2016.