Non-Newtonian Flow with Heat and Mass Transfer over a Porous Stretching Sheet Under the Effects of Space Dependent and TGD Heat Sink, Non - Uniform Heat Source and Dissipation of Energy and CGD Mass Diffusion with Suction/Blowing

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"NON-NEWTONIAN FLOW WITH HEAT AND MASS TRANSFER OVER A POROUS STRETCHING SHEET UNDER THE EFFECTS OF SPACE DEPENDENT AND TGD HEAT SINK, NON - UNIFORM HEAT SOURCE AND DISSIPATION OF ENERGY AND CGD MASS DIFFUSION WITH SUCTION/BLOWING"

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Abstract

In the present paper, an analytical study of visco - elastic fluid flow with heat and mass transfer characteristics over a porous stretching sheet has been examined. Heat balance is maintained with space dependent and temperature gradient dependent heat sink/source, viscous dissipation, and non-uniform heat source on the non-Newtonian Walter's liquid B' model. The mass balance is maintained with Chemically reactive species of order I, variable mass diffusivity and concentration gradient dependent mass diffusion. Using similarity transformation technique on the highly non-linear differential equations,

several closed form analytical solutions are obtained for non-dimensional temperature and concentration in both PST and PHF cases in the form of confluent hypergeometric (Kummar's) functions. The effect of permeability parameter, viscoelastic parameter, suction parameter on velocity profiles and various physical fluid situations with different degrees of viscoelasticity, Prandtl number, Eckert number, heat source- sink strength, temperature field are discussed in detail and presented through graphs. Similarly in the mass transfer field the effects of Schmidt number, reaction rate parameters are discussed in detail and presented through graphs.

Key Words: Concentration Gradient Dependent (CGD), Flow and Heat Transfer, Non-Newtonian Flow, Porous Medium, Stretching Sheet, Temperature Gradient Dependent (TGD), Visco-elastic Fluid

1. Introduction

Many practical applications, such as food stuff, polymer, molten plastic, blood, fluids are non-Newtonian in their flow characteristics. Boundary layer flow over a continuous moving/stretching surface is an important type of flow occurring in a number of engineering processes. Cooling of a metallic plate in a cooling bath, Aerodynamic extrusion of plastic and rubber sheets, which may be an electrolyte, crystal glowing, the boundary layer along a liquid film in condensation process and filament or polymer sheet extruded continuously from a dye. Various aspects of this problem have been investigated by Sakiadis [1-3]. Erickson et al. [4] studied the temperature distribution in the boundary layer flow with suction or blowing and relevant experimental results were studied by Tsou et al. [5] including many aspects for the flow and heat transfer boundary layer flow problems past a continuous moving plate. Non-Newtonian fluid flow which are viscoelastic in nature past a stretching sheet was examined by Siddappa and Khapate [6]. Dutta and Gupta [7] have studied the analytical solution for the velocity field and temperature distribution over the stretching sheet.

Rajagopal et al. [8] studied the same flow as in Ref [6] and obtained a similarity solution of the boundary layer equations numerically for the case of small visco-elastic parameter. In many practical applications there exists significant temperature difference between the surface and the ambient fluid. In this regard Vajravelu and Nayfeh [9] have studied the effect of temperature dependent heat source /sink in heat transfer characteristics.

The study of the temperature field with modified generation or absorption of heat in a moving fluid is important in view of many physical situations-such as (i) Problems concerned with dissociating fluids. (ii) Problems dealing with chemical reactions.

The heat transfer in a visco -elastic fluid past a stretching sheet with viscous dissipation and internal heat generation have been studied by Veena et al. [10] and the flow of visco elastic fluid obeying Walter's liquid B' model past a stretching sheet with and without suction have been analysed by Abel [11,12] and also attempted an exact solution of the boundary layer equation of motion.

Similar solutions of laminar boundary layer equations describing the study of two-dimensional flow and heat transfer in an electrically conducting and heat generating fluid driven by a continuous moving porous surface immersed in a fluid saturated porous medium has been investigated Ali Chamkha [13]. A space dependent exponentially decaying heat generation or absorption is considered on flow and heat transfer past a vertical plate by [14-15].

The effect of suction and blowing on the heat transfer problem in the presence of the porous media is reported by a smaller number of researchers. The conjugate heat transfer problem from an accelerating surface in presence of uniform suction and blowing in a viscous fluid has been solved by Dutta [16]. Chen and Char [17] made an analysis about the effects of PST (prescribed surface temperature) and PHF (Prescribed wall heat flux) boundary conditions on the heat transfer characteristics of a continuous linearly stretching sheet subjected to suction or blowing.

The heat and mass transfer in a visco- elastic fluid flow over an accelerating with heat source and viscous dissipation for both the PST and PHF cases has been done by Sonth *et.al*. [18]. K.Rajagopal et.al.[19] were studied the diffusion of chemically reactive species of an electrically conducting visco elastic fluid immersed in a porous medium over a stretching sheet with PST and PHF cases.

Motivated by all the above analyses in the present paper, an analytical study of visco -elastic fluid flow with heat and mass transfer characteristics for prescribed surface temperature boundary conditions (PST) and prescribed wall flux (PHF) in heat transfer and Prescribed power law surface concentration (PST) in mass transfer over a porous stretching sheet have been studied.

2. Formulation of the problem

Study of two-dimensional boundary layer flow of an incompressible visco elastic fluid flow of the type Walter's liquid B' model over a stretching surface is considered for investigation. The flow is generated due to the stretching of the sheet by applying two equal and opposite forces along the x-axis and keeping the origin fixed. The flow is assumed to be confined in a region y>0. It is also studied that the fluid considered for analysis is of non-Newtonian type. The basic boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\,\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\,\partial y}+\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial x}\right\}-\frac{\nu}{k'}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)= k\frac{\partial^2 T}{\partial y^2}+q_{\eta\eta\eta}+Q(T-T_\infty)+\mu\left(\frac{\partial u}{\partial y}\right)^2+Q'\frac{\partial T}{\partial y}+\rho k_0\frac{\partial u}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}+k_r(C-C_\infty)+D\frac{\partial c}{\partial y} \tag{4}$$

Where u and v are the flow velocities in x and y directions respectively. k_0 is the coefficient of elasticity, ν is the kinematic viscosity and k' denotes permeability , $q_{\eta\eta\eta}$ is the space dependent internal heat generation /absorption or rate of internal heat generation, ρ is the fluid density,c_p is the specific heat at constant pressure, μ is the dynamic viscosity, Q' is the volumetric rate of heat absorption, k_r denotes reaction rate coefficient, k is the thermal conductivity, Q is the specific internal heat generation or absorption and D is mass diffusion coefficient.

The term $q_{\eta\eta\eta}$ is modelled as

$$q_{\eta\eta\eta}=\frac{k u_w(x)}{x\nu}\left(A^*(T_w-T_\infty)e^{-\alpha\eta}+B^*(T-T_\infty)\right)$$

Where T_w is the temperature at the wall, T_∞ is the fluid temperature far away from the surface, k is the thermal conductivity and A* and B* are the parameters of space dependent and temperature-dependent internal heat generation (i.e., A* > 0 and B* > 0)/absorption (i.e., A* < 0 and B* < 0), respectively.

The appropriate boundary conditions on velocity field take the form

$$u=cx,\qquad v=v_w\qquad \text{at}\qquad y=0$$

$$u=0,\qquad u_y\to 0\qquad \text{as}\qquad y\to\infty \tag{5}$$

Here suffix y represents differentiation w.r.t. y, c is the constant know as stretching rate and v_w is the suction velocity across the stretching sheet when $v_w<0$ and it is blowing velocity $v_w>0$

3. Solution of the Momentum Equation:

To solve momentum equation (2), defining the new velocity variables as follows

$$u=cx\, f_\eta(\eta), \quad v = -\sqrt{c\nu}\, f(\eta), \quad \eta=\sqrt{\frac{c}{\nu}}\, y \tag{6}$$

where f is the dimension less stream function and η is the similarity variable. Substitution of equation (6) in equation (2) results in a fourth order non-linear differential equation in non-dimensional form

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} k_1 \{2f_\eta f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\} - k_2 f_\eta \tag{7}$$

Where,

$k_1 = \frac{k_0 c}{\nu}$ - is the visco- elastic parameter

$k_2 = \frac{\nu}{k' c}$ - is the permeability parameter

The corresponding boundary conditions on f take the form

$$f = -\frac{v_w}{\sqrt{c\nu}}, \quad f_\eta(0) = 1 \quad \text{at} \quad \eta = 0$$

$$f_\eta(\eta) = 0, \quad f_{\eta\eta}(\eta) = 0 \quad \text{as} \quad \eta \to \infty \tag{8}$$

Now exact solution of equation (7) with B.Cs (8) is of the form

$$f(\eta) = a - \left(\frac{1}{b}\right) e^{-\alpha\eta} \tag{9}$$

Where,

$$a = \frac{1}{2(1+k_2)}\left[R(1 + 2k_2) + \sqrt{R^2 + 4(1 + k_2)}\right]$$

$$b = \frac{1}{2}\left(R + \sqrt{R^2 + 4(1 + k_2)}\right) \tag{10}$$

$$\alpha = \sqrt{\frac{(1+k_2)}{(1-k_1)}}$$

$R = \frac{v_w}{\sqrt{c\nu}}$ - is a suction parameter.

Where α is a real positive root of the fallowing cubic equation and solved by Graffe's root square method.

$$\alpha^3 - \frac{1}{k_1 a}\alpha^2 + \frac{1}{k_1}\alpha + \frac{k_2}{k_1 a} = 0 \tag{11}$$

4. Heat Transfer Analysis:

4.1 PST Case (Prescribed Surface Temperature):

Solution of the heat transfer equation (3) depends on the nature of the prescribed boundary conditions.

The appropriate boundary conditions on temperature are defined in PST case as

$$T_w = T_\infty + A_1\left(\frac{x}{l}\right)^2 \quad \text{at} \quad y \to 0$$
$$T \to T_\infty \quad \text{as} \quad y \to \infty \tag{12}$$

Where A_1 is a constant whose value depends on the properties of the fluid and l is the characteristic length.

And defining the non-dimensional temperature variable $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{13}$$

To obtain similarity solutions for temperature $\theta(\eta)$, stretched boundary surface with prescribed power law temperature of second degree is considered. Using transformation (6) and (13), heat equation (3) becomes,

$$\theta_{\eta\eta} + \left(\Pr f(\eta)\theta_\eta + Q^*\right)\theta_\eta(\eta) - \Pr\left[2f_\eta(\eta) - \frac{B^*}{\Pr} - \frac{\beta}{\Pr}\right]\theta(\eta) = -A^* e^{-\alpha\eta} - E_c f_{\eta\eta}^2(\eta) - E_c^*\left[f_\eta^2(\eta) f_{\eta\eta}^2(\eta) - f(\eta) f_{\eta\eta}^2(\eta)\right] \tag{14}$$

Where,

$\Pr = \frac{\rho c_p \nu}{k}$ - is the Prandtl number

$\beta = \frac{\nu}{kc} Q$ - is the Source /Sink parameter

$E_c = \frac{\mu c^2 \nu l^2}{k A_1}$ - is the Eckert number

$E_c^* = \frac{\rho k_0}{k A_1} c^2 \sqrt{\frac{c}{\nu}}\, l^2 \nu$ - is the modified Eckert number

$Q^* = \frac{\nu}{kc}\sqrt{\frac{c}{\nu}}\, Q$ - Non-dimensional constant heat source

And the corresponding boundary conditions (12) transform to dimensionless

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{15}$$

Defining the change of variable

$$\xi = -\frac{\Pr}{\alpha^2} e^{-\alpha\eta} \tag{16}$$

And making use of the equations (9), (10) and (16) in equation (14), the governing non dimensional form of energy equation is obtained in terms of ξ as follows

$$\xi\theta_{\xi\xi} + [1 - (p_0 - Q^{**}) - \xi]\theta_\xi + \left[2 + \frac{1}{\xi\alpha^2}(B^* + \beta)\right]\theta = \frac{A^*}{\Pr} - (E_c + E_c^*)\frac{\alpha^6}{\Pr^2 b^2}\xi \tag{17}$$

Where,

$$p_0 = \frac{\Pr}{\alpha^2}$$

$Q^{**} = \frac{Q^*}{\alpha^2}$ - Modified Non-dimensional constant heat source

The corresponding boundary conditions (15) convert to

$$\theta\left(-\frac{\Pr}{\alpha^2}\right) = 1, \theta(0) = 0 \tag{18}$$

The solution of the equation (17) subject to the boundary conditions in (18) is obtained in the following form of confluent hypergeometric functions of the similarity variable η as

$$\theta(\eta) = \left[1 - C_1\frac{\Pr^2}{\alpha^2}\right](e^{-\alpha\eta})^{\frac{p_0+q_0}{2}} \frac{M\left[\frac{p_0+q_0-4}{2}, \ 1+q_0, -\frac{\Pr^2}{\alpha^2}e^{-\alpha\eta}\right]}{M\left[\frac{p_0+q_0-4}{2}, \ 1+q_0, -\frac{\Pr^2}{\alpha^2}\right]} - C_1\left[-\frac{\Pr}{\alpha^2}e^{-\alpha\eta}\right]^2 \tag{19}$$

Where,

$$C_1 = \frac{\frac{-A^*}{\Pr\xi} - \frac{(E_c+E_C^*)}{\Pr^2 b^2} a\,\alpha^6}{4 - 2p_0 + \frac{1}{\alpha^2}(B^*+\beta)}$$

And $$q_0 = \sqrt{(p_0 + Q^*) - 4\frac{(B^*+\beta)}{\alpha^2}} \tag{20}$$

Further Dimensionless wall temperature gradient $\theta_\eta(0)$ is derived and obtained as

$$\theta_\eta(0) = A_0 \left\{ \frac{Pr}{2\alpha} \left[\frac{p_0+q_0-4}{1+q_0} \right] M \left[\frac{p_0+q_0-2}{2}, 2+q_0, -\frac{Pr}{\alpha^2} \right] - \alpha \left(\frac{p_0+q_0}{2} \right) M \left[\frac{p_0+q_0-4}{2}, 1+q_0, -\frac{Pr}{\alpha^2} \right] - C_1 \left(-\frac{Pr}{\alpha^2} \right)^2 \right\} \quad (21)$$

Where,

$$A_0 = \frac{\left(1 - C_1 \frac{Pr^2}{\alpha^4}\right)}{\left(\frac{p_0+q_0-4}{2}, 1+q_0, -\frac{Pr}{\alpha^2}\right)}$$

Taking into account the thermal radiation and temperature gradient dependent heat sink, the local heat flux is defined and expressed as follows

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_w = k\sqrt{\frac{c}{v}}(T_w - T_\infty)\left(-\theta_\eta(0)\right) \quad (22)$$

4.2 PHF Case (Prescribed Wall Flux):

In PHF case, solution of the heat transfer equation (3) depend on the nature of the prescribed Power law heat flux boundary conditions which are defined as

$$-k\left(\frac{\partial T}{\partial Y}\right) = q_w = E\left(\frac{x}{l}\right)^2 \quad \text{at} \quad y = 0 \quad (23)$$

$$T \to T_\infty \quad \text{as} \quad y \to \infty \quad (24)$$

Where E is constant

Defining a new variable as

$$g(\eta) = \frac{T - T_\infty}{\frac{E}{k}\sqrt{\frac{v}{c}}\left(\frac{x}{l}\right)^2} \quad (25)$$

and using the equations (6), (9) and (13) in equation (3) it deduces to the following non dimensional boundary layer equation as

$$g_{\eta\eta} + (Prf(\eta) + Q^{**})g_\eta(\eta) - Pr\left[2f_\eta(\eta) - \frac{B^*}{Pr} - \frac{\beta}{Pr}\right]g(\eta) = -A^* e^{-\alpha\eta} - E_c f_{\eta\eta}^2(\eta) - E_c^*\left[f_\eta^2(\eta) f_{\eta\eta}^2(\eta) - f(\eta) f_{\eta\eta}^2(\eta)\right] \quad (26)$$

And the corresponding boundary conditions transform to

$$g_\eta(0) = -1, \qquad g(\infty) = 0 \tag{27}$$

Following the same solution methodology as done in PST case for PHF case also. The solution of equation (26) is obtained as fallows

$$g(\eta) = \left(\frac{-pr}{\alpha^2}\right)^{\frac{p_0+q_0}{2}} \left[(e^{-\alpha\eta})^{\frac{p_0+q_0}{2}} \left[\frac{p_0+q_0-4}{2(1+q_0)}\right] M\left(\frac{p_0+q_0-4}{2}, 1+q_0, \frac{-pr}{\alpha^2} e^{-\alpha\eta}\right)\right] + C_1^*\left(\frac{-pr}{\alpha^2} e^{-\alpha\eta}\right)^2 \tag{28}$$

Where,

$$C_1^* = \frac{\alpha(1+C_2)}{2(-pr)^2} \tag{29}$$

$$C_2 = \left(\frac{-pr}{\alpha^2}\right)\frac{p_0+q_0}{2}\left\{\frac{p_0+q_0-4}{2(1+q_0)} M\left(\frac{p_0+q_0-2}{2}, 2+q_0, \frac{-pr}{\alpha^2}\right)\left(\frac{-pr}{\alpha}\right) - \alpha\left(\frac{p_0+q_0}{2}\right) M\left(\frac{p_0+q_0-4}{2}, 1+q_0, \frac{-pr}{\alpha^2}\right)\right\} \tag{30}$$

And the temperature at sheet is given by

$$T_w = T_\infty + \frac{Ex^2}{k}\sqrt{\frac{\nu}{c}}\, g(0) \tag{31}$$

Where the dimensionless wall temperature is

$$g(0) = \left(\frac{-pr}{\alpha^2}\right)^{\frac{p_0+q_0}{2}}\left[\left[\frac{p_0+q_0-4}{2(1+q_0)}\right] M\left(\frac{p_0+q_0-4}{2}, 1+q_0, \frac{-pr}{\alpha^2}\right)\right] + C_1^*\left(\frac{-pr}{\alpha^2}\right)^2 \tag{32}$$

5. Mass Transfer Analysis:

5.1 Solution Methodology:

Solution of the mass transfer equation (4) depend on the nature of the prescribed boundary conditions

The appropriate boundary conditions on concentration are considered as

$$C = C_w = C_\infty + A_2\left(\frac{x}{l}\right)^2 \qquad \text{at} \qquad y = 0$$

$$C \rightarrow C_\infty \quad \text{as} \quad y \rightarrow \infty \tag{33}$$

Where, C_w denotes the concentration at the wall and C_∞ denotes the concentration far away from the wall. A_2 is constant whose values depends on the properties of the fluid and l is the characteristic length.

And defining the non-dimensional concentration variable as

$$\varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{34}$$

To obtain the similarity solutions for concentration $\varphi(\eta)$, we considered stretched boundary surface with prescribed power law concentration of second degree only. Using second transformation (6) and (34), equation (4) becomes

$$\varphi_{\eta\eta}(\eta) + (Sc\, f(\eta) + M^*)\varphi_\eta(\eta) - 2Scf_\eta(\eta)\varphi(\eta) = \beta^* Sc\varphi(\eta) \tag{35}$$

Where,

$Sc = \frac{\nu}{D}$ - Schmidt number

$M^* = \frac{\nu}{c}\sqrt{\frac{c}{\nu}}$ - Non dimensional constant mass source

$\beta^* = \frac{k_r}{c}$ - reaction rate parameter

Using equation (34) in equation (33), we get the boundary conditions in dimensionless form as

$$\varphi(0) = 1, \quad \varphi(\infty) = 0 \tag{36}$$

and f is given in equation (9)

Clearly the concentration field φ is coupled to the velocity field through the dimensionless stream function f in the non-linear concentration equation (35)

In the special case of non-reacting species ($\beta^* = 0$) the non-linear term on the right-hand side of equation (35) vanishes and present concentration boundary layer equation becomes

$$\varphi_{\eta\eta}(\eta) + (Sc\, f(\eta) + M^*)\varphi_\eta(\eta) - 2Scf_\eta(\eta)\varphi(\eta) = 0 \tag{37}$$

And the corresponding boundary conditions are

$$\varphi(0) = 1, \quad \varphi(\infty) = 0 \tag{38}$$

defining the change of variable

$$\xi = -\frac{Sc}{\alpha^2} e^{-\alpha\eta} \tag{39}$$

and making use of the equation (9), (10) and (39) in equation (37), We obtained the governing non dimensional form of concentration equation in terms of ξ as follows

$$\xi\varphi_{\xi\xi} + (1 - C_0 - \xi)\varphi_\xi(\xi) + \frac{2\alpha}{b}\varphi(\xi) = 0 \tag{40}$$

where,

$$C_0 = \frac{Sc}{\alpha^2} ab - \frac{M^*}{\alpha^2} b\nu$$

and the corresponding boundary conditions are

$$\varphi\left(-\frac{Sc}{\alpha^2}\right) = -1, \qquad \varphi(0) = 0 \tag{41}$$

The solution of the equation (40) subjected to the boundary conditions (41) is obtained in the following form of similarity variable η as

$$\varphi(\eta) = c\left(-\frac{Sc}{\alpha^2} e^{-\alpha\eta}\right)^{c_0} M(s^* - c_0\,,\ 1 + c_0,\ \xi) \tag{42}$$

where,

$$c = \left[c\left(-\frac{Sc}{\alpha^2} e^{-\alpha\eta}\right)^{c_0} M\left(s^* - c_0\,,\ 1 + c_0, -\frac{Sc}{\alpha^2}\ \right)\right]^{-1} \tag{43}$$

Further dimensionless wall concentration gradient $\varphi_\eta(0)$ is derived and obtained as

$$\varphi_\eta(0) = c\left(-\frac{Sc}{\alpha^2}\right)^{c_0}\left[\frac{sc}{\alpha}\left(\frac{s^* - c_0}{1 + c_0}\right) M\left(s^* - c_0 + 1\,,\ 2 + c_0,\ -\frac{Sc}{\alpha^2}\right) + \alpha c_0\left(s^* - c_0 + 1\ ,\ 2 + c_0,\ -\frac{Sc}{\alpha^2}\ \right)\right] \tag{44}$$

The mass transfer analysis would be carried out by analysing the terms of the local mass flux which defined as

$$m_w = -D\left(\frac{\partial c}{\partial y}\right)(C_w - C_\infty)[-\varphi_\eta(0)] \tag{45}$$

6. Results and Discussion:

In this investigation the visco-elastic fluid flow over a stretching sheet in a saturated porous medium is considered. the basic equations of momentum and heat transfer are highly non-linear and converted into a set of ordinary differential equations. The solution of governing energy equation is obtained in terms of confluent hyper geometric functions (Kummar's function).

Thus, the solutions for the heat transfer problem with space dependent and temperature gradient dependent heat sink/source is presented. In order to derive the several closed form analytical solutions. Also, the effect of permeability parameter, visco-elastic parameter, suction parameter on velocity profile and various physical fluid situations are discussed in detail and presented through graphically.

Figure 1a is the representation of $f_\eta(\eta)$ Vs η for different values of porosity parameter k_2 and for zero suction. From the Figure it is noticed that the increasing values of k_2 and the suction parameter decrease the velocity distribution in the flow field.

Figure 1b is the plot of velocity profile $f_\eta(\eta)$ Vs η for various values of visco elastic parameter k_1. This figure physically implies that the rise in values of visco-elastic parameter k_1 decreases the velocity profile.

Figure 1c is the representation of $f_\eta(\eta)$ Vs η for different values of porosity parameter k_2 and suction parameter v_w. From the figure it is noticed that decrease the velocity distribution in the flow field while increasing value of k_2 and v_w.

Figure 1d is the depict of velocity profile $f_\eta(\eta)$ Vs η for different values of suction/blowing parameter v_w and also k_2. From the figure it is observed that an increasing values of v_w and k_2 is to decrease the velocity profile.

Figure 2a is the representation of the effect of time dependent parameter B* on temperature profile θ(η) in PST case. The graph is physically implying that the increase of time-dependent parameter B* results in the increase in temperature distribution for increasing values of visco-elastic parameter k_1.

Figure 2b is the graph of temperature distribution θ(η) Vs η for various values of Eckert number Ec and suction parameter v_w in PST case. From the figure it is depicted that the effect of increasing values of Ec is to increase the temperature distribution for $v_w>0$.

Figure 2c is drawn to display the nature of temperature profile $\theta(\eta)$ Vs η. The effect of heat source/sink β and suction parameter v_w on $\theta(\eta)$ is shown in this figure. The graph is physically noticed that the decrease the temperature distribution for v_w=-0.235 while increasing values of heat source parameter β.

Figure 2d is drawn to display the nature of temperature distribution $\theta(\eta)$ Vs η for different values of suction/blowing parameter v_w in PST case. This graph demonstrates that the temperature distribution decreases with increasing values of v_w.

Figure 2e represents the graph of $\theta(\eta)$ Vs η for various values of v_w in PST case. From This graph noticed that temperature profile decreases with increasing the values of v_w. It is also noticed that due to the effect of v_w slight characteristics of dissipation and blowing exhibited on the graph in temperature distribution for v_w=0 and v_w=0.235

Figures (3a-3d), the plots of temperature profile $g(\eta)$ Vs η for different sets of parameters are drawn in PHF case. From the figures it is noticed that the wall temperature is changed while changing the physical parameters like the visco- elastic parameter (k_1), Prandtl number (Pr), space dependent term (A^*) and temperature dependent term (B^*). Also, it is noticed that the wall temperature is not unity. From the figs (3b and 3c) it is clearly found that temperature of the sheet is highly increased for greater value of A^* and B^* (i.e., A^*=1 and B*=0.1).

Figures (4a) and (4b) represents the graphs of the non-dimensional concertation profile $\Phi(\eta)$ Vs η for various values of Schmidt number Sc. The concertation profile in the flow filed decreases while increasing the value of Schmidt numbers Sc i.e. a decrease of molecular diffusivity D, it results in a decrease of the concertation boundary layer. Hence the concertation of the species is lower for larger values of Sc and larger for small values of Sc. And also, we notice that the effect of Suction parameter v_w is to decrease the concentration profile.

The both Schmidt number (Sc) and suction parameter v_w (v_w >0) effects that to reduce the concertation profile.

Comparing both the figs 4a and 4b it results that the concertation profile g(n) increases in the presence of visco-elastic parameter k_1.

Figures (4c) and (4d) represents the graphs of the dimensionless concentration profile $\Phi(\eta)$ Vs η for various values of β is to reduces the concentration boundary thickness and increases the mass transfer even though in the presence of porous medium. This is in agreement with the equivalent heat transfer problem for which numerical data of Veena and Abel [20] have been plotted in the figures

7. Graphs

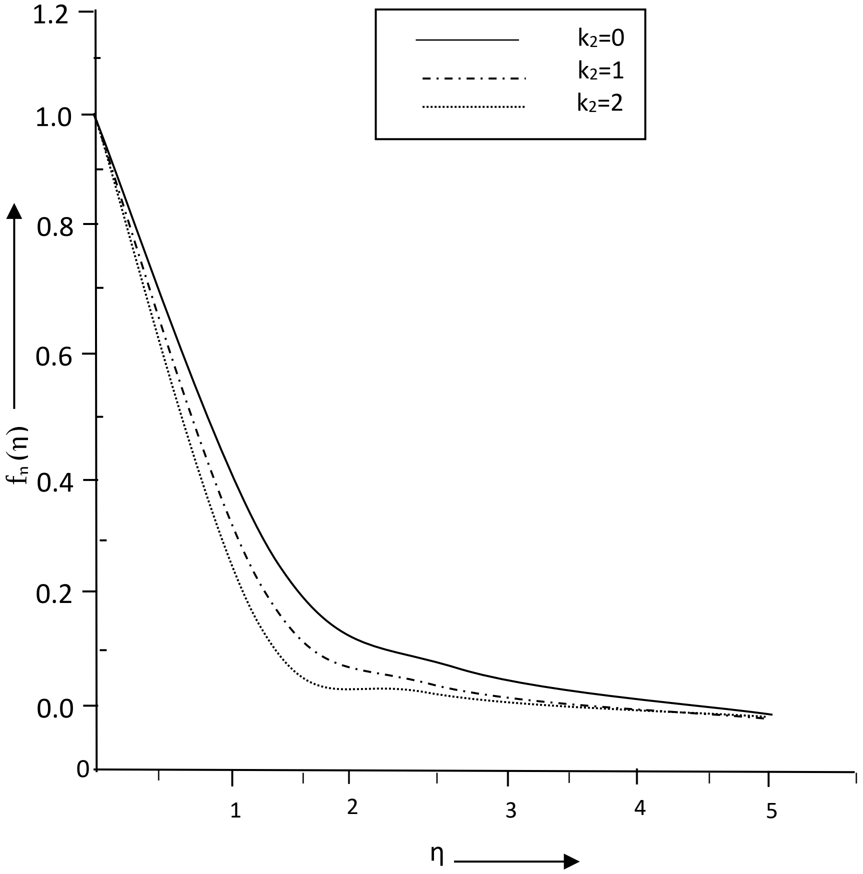

FIGURE (1a): THE GRAPH OF VELOCITY PROFILE $f_{\eta}(\eta)$ Vs η FOR DIFFERENT VALUES OF k_2 = 0, 1, 2 WITH V_w = 0, k_1 = 0.2

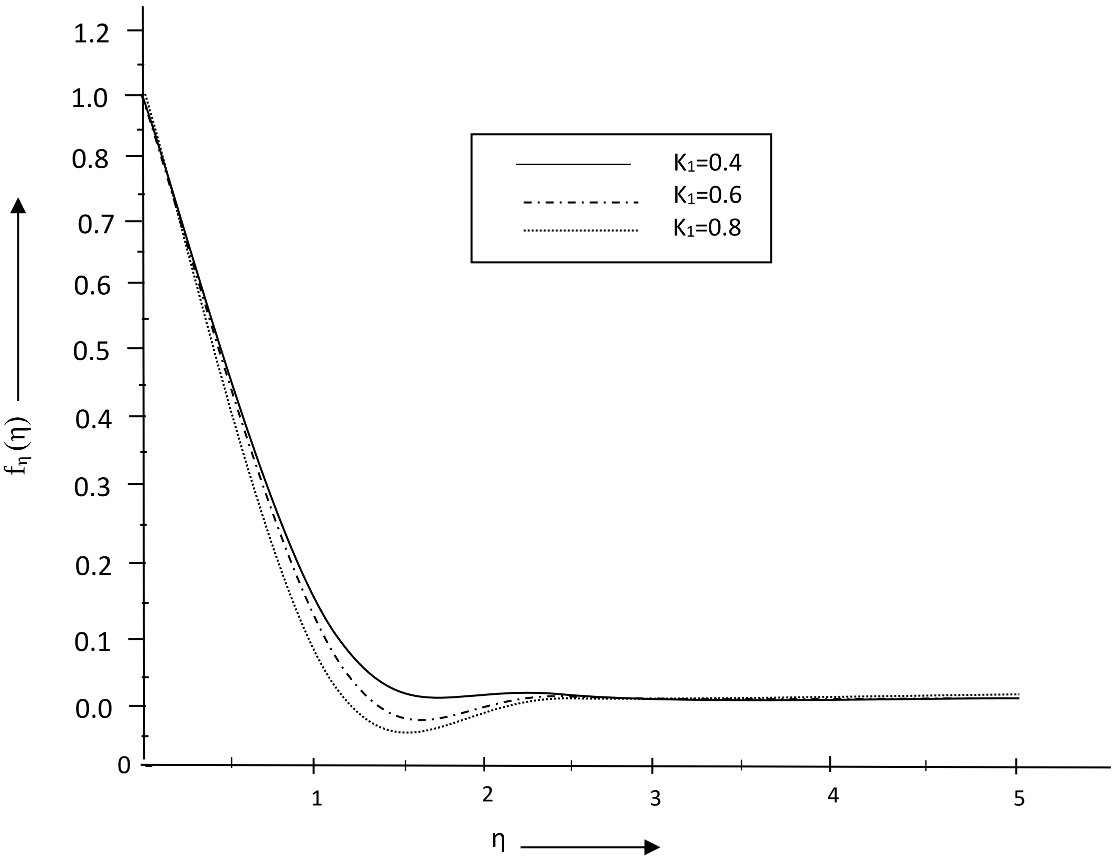

FIGURE (1b): THE GRAPH OF VELOCITY PROFILE $f_\eta(\eta)$ Vs η FOR DIFFERENT VALUES OF k_1 = 0.4, 0.6, 0.8 WITH k_1 = 1, V_w = 0

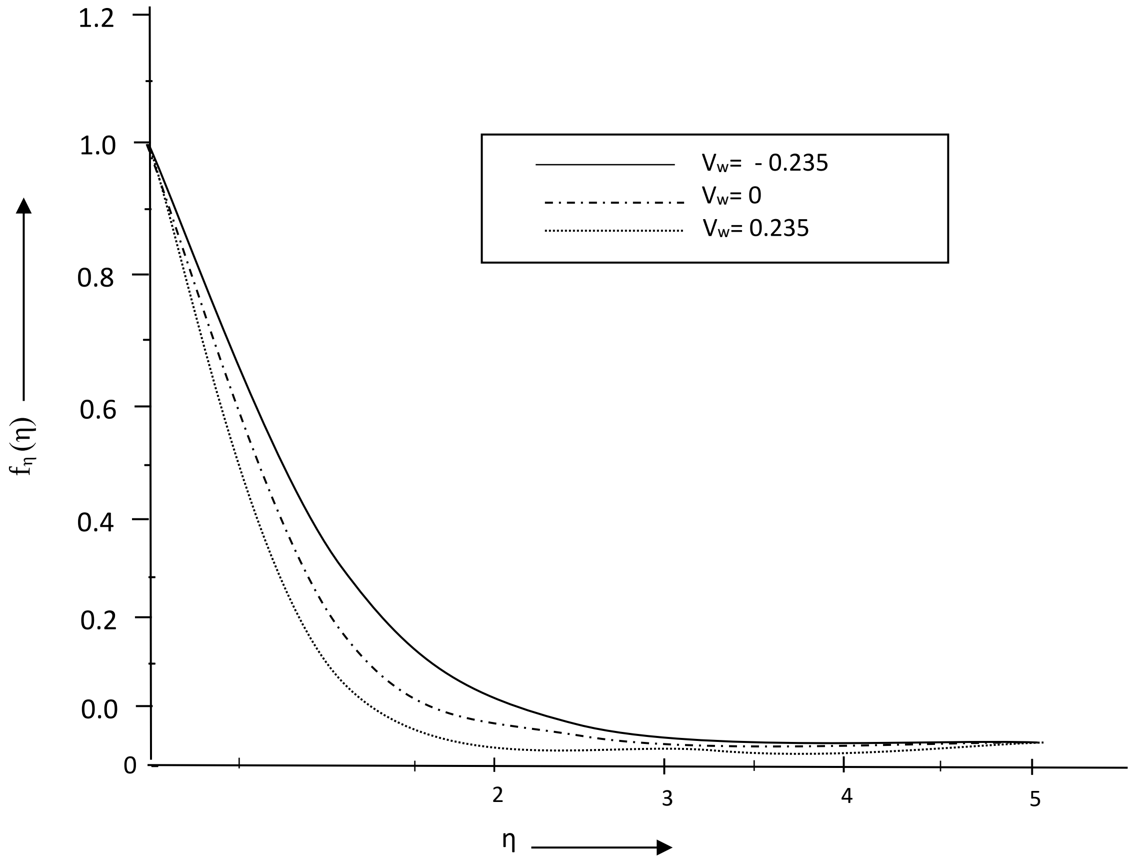

FIGURE (1c): THE GRAPH OF VELOCITY PROFILE $f_\eta(\eta)$ Vs η FOR DIFFERENT VALUES OF V_w=-0.235, 0, 0.235 WITH k_1 = 0.2, k_2 =1.0

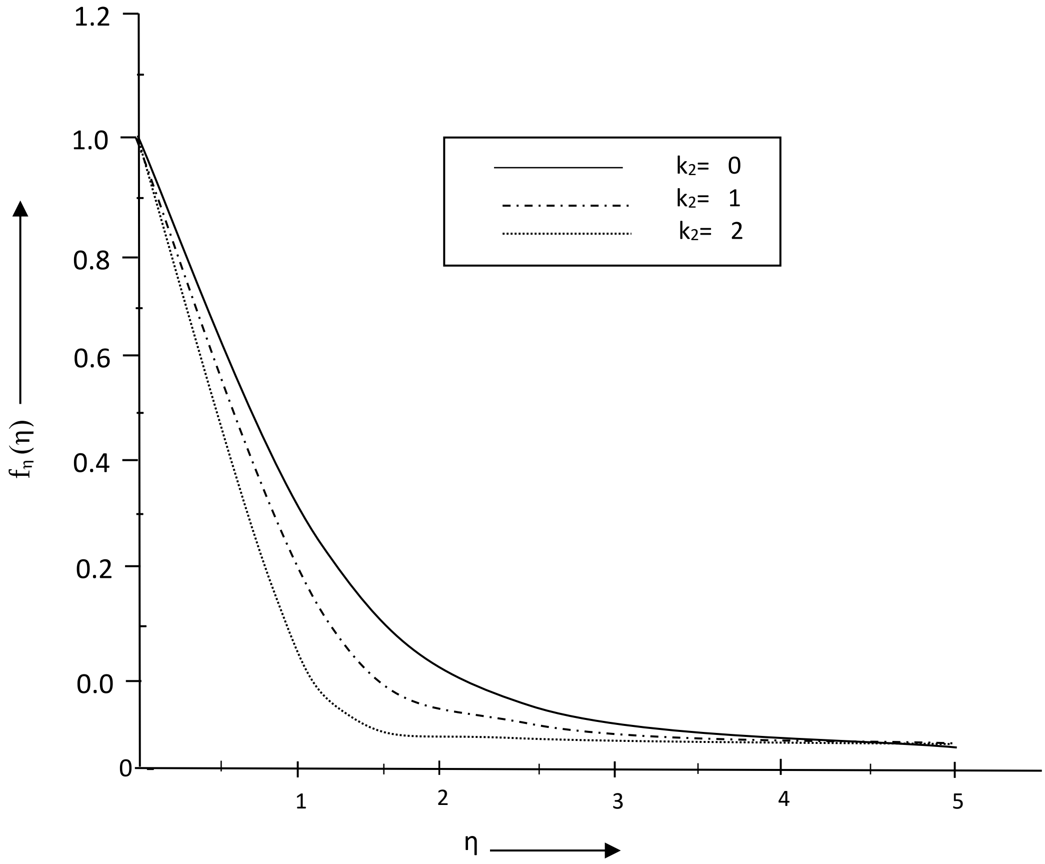

FIGURE (1d): THE GRAPH OF VELOCITY PROFILE f $_\eta(\eta)$ Vs η FOR DIFFERENT VALUES OF k_2= 0, 1, 2 v_w = - 0.235, 0, 0.235 WITH k_1 = 0.2

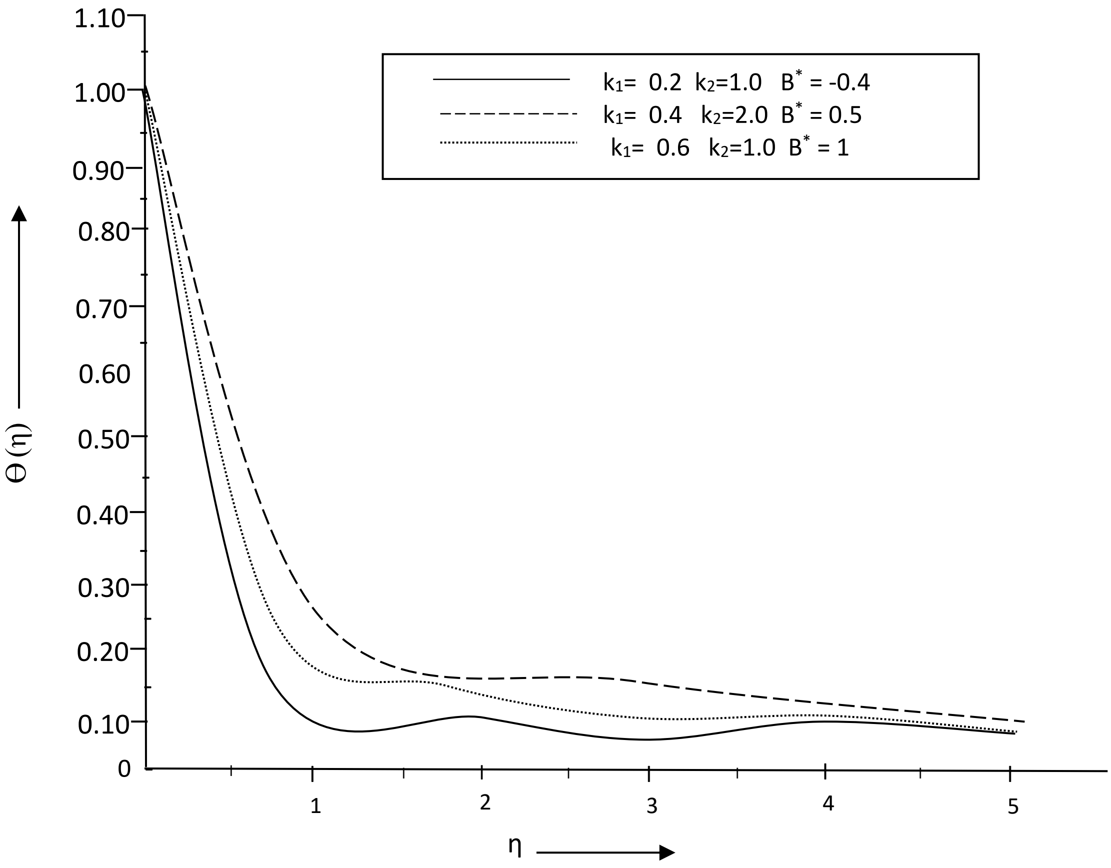

FIGURE (2a): THE GRAPH OF TEMPERATURE PROFILE Θ (η) Vs η FOR VARIOUS VALUES OF k1, k2 AND B* WITH β=-0.1, Ec = 0.2 AND Ec/ = 0.24

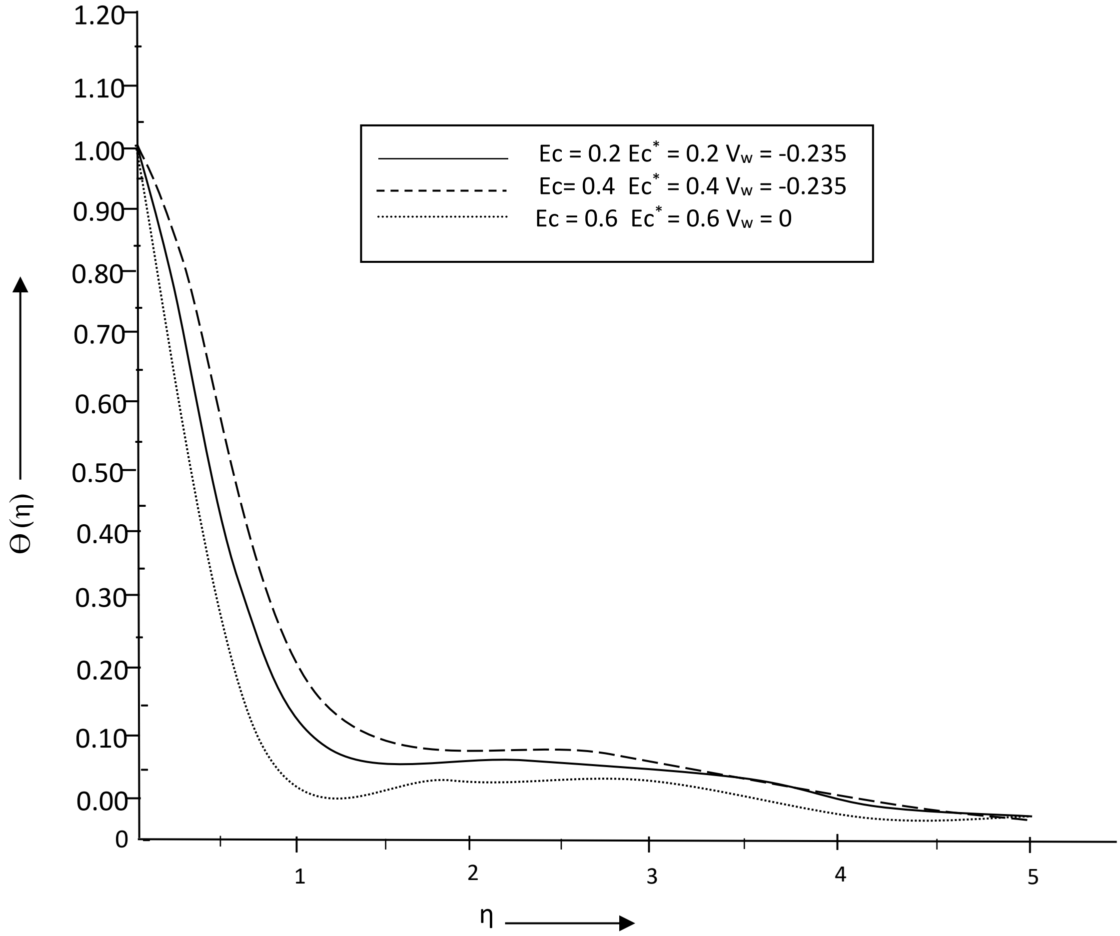

FIGURE (2b): THE GRAPH OF TEMPERATURE PROFILE Θ (η) Vs η FOR VARIOUS VALUES OF Ec, Ec^* AND v_w WITH β = - 0.1, Pr = 0.71, k_2 = 1.0 AND k_1= 0.1

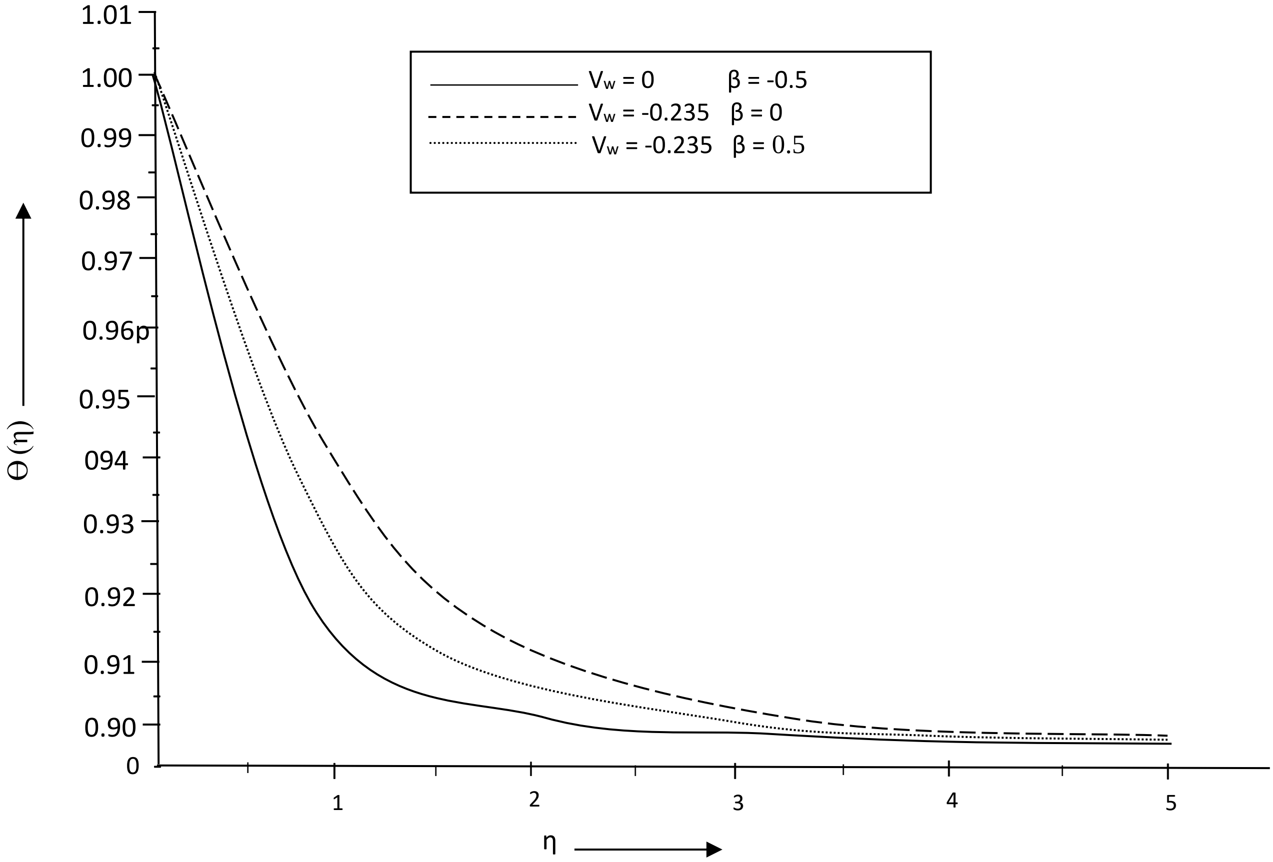

FIGURE (2c): THE GRAPH OF TEMPERATURE PROFILE Θ (η) Vs η FOR VARIOUS VALUES OF v_w AND β WITH Ec^* =0.24 AND Ec = 0.2 , Pr = 0.71, k_2 = 1.0 AND k_1= 0.2

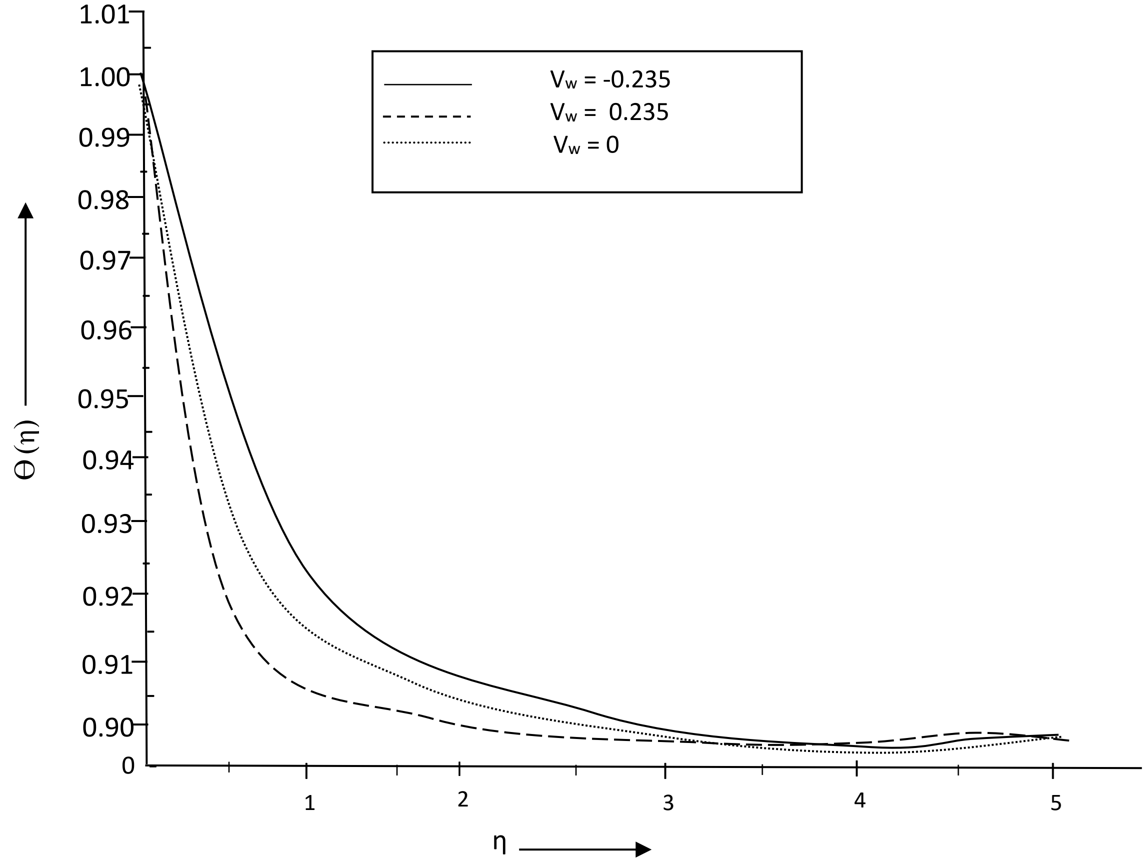

FIGURE (2d): THE GRAPH OF TEMPERATURE PROFILE Θ (η) Vs η FOR DIFFERENT VALUES OF v_w WITH Pr = 0.71,β = -0.1WITH Ec^*=0.24 AND Ec = 0.2

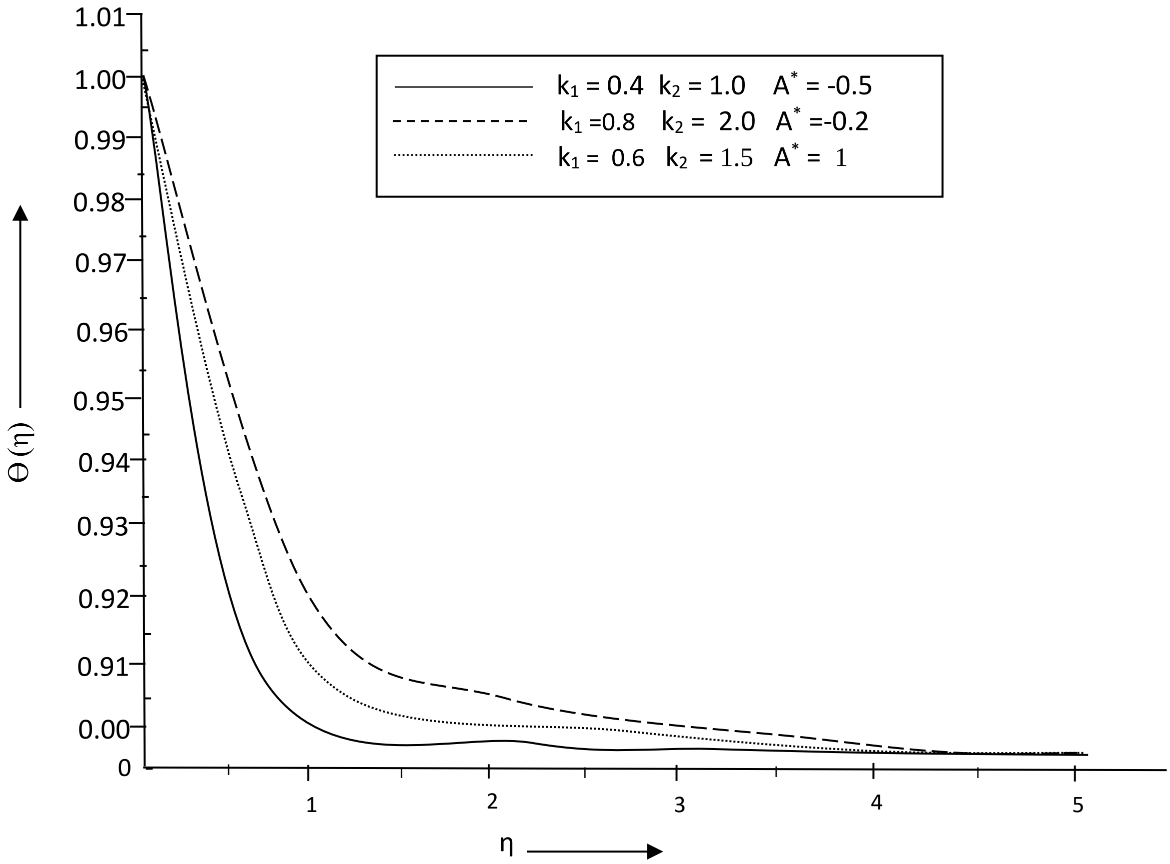

FIGURE (2e): THE GRAPH OF TEMPERATURE PROFILE Θ (η) Vs η FOR VARIOUS VALUES OF k_1, k_2 and A^* WITH Pr = 0.71, β = -0.1

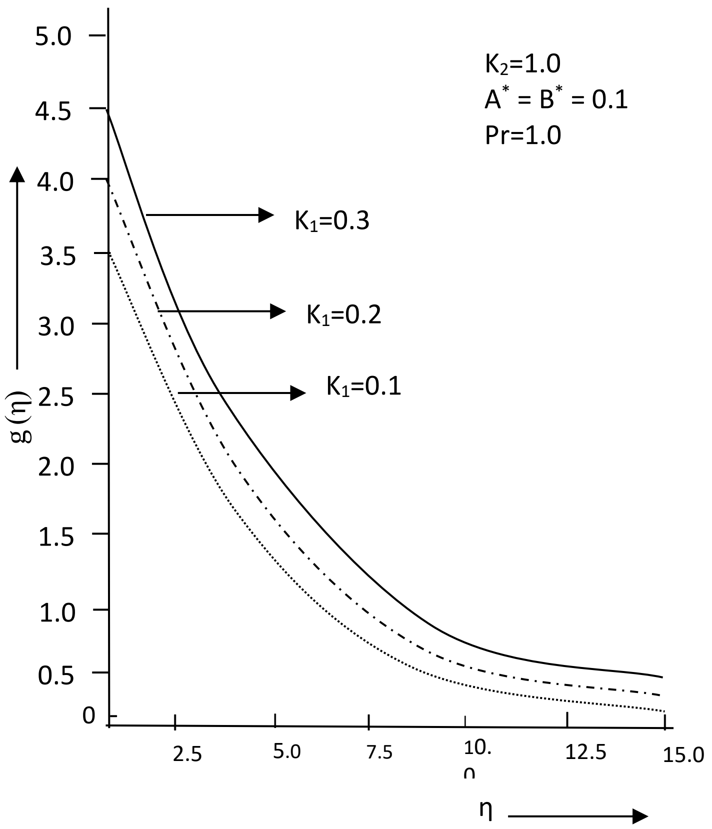

FIGURE (3a): THE GRAPH OF TEMPERATURE PROFILE g(η) Vs η FOR DIFFERENT VALUES OF k_1 = 0.3, 0.2, 0.1 WITH k_2=1.0 , A^* = B^* = 0.1, Pr=1.0 IN PHF CASE

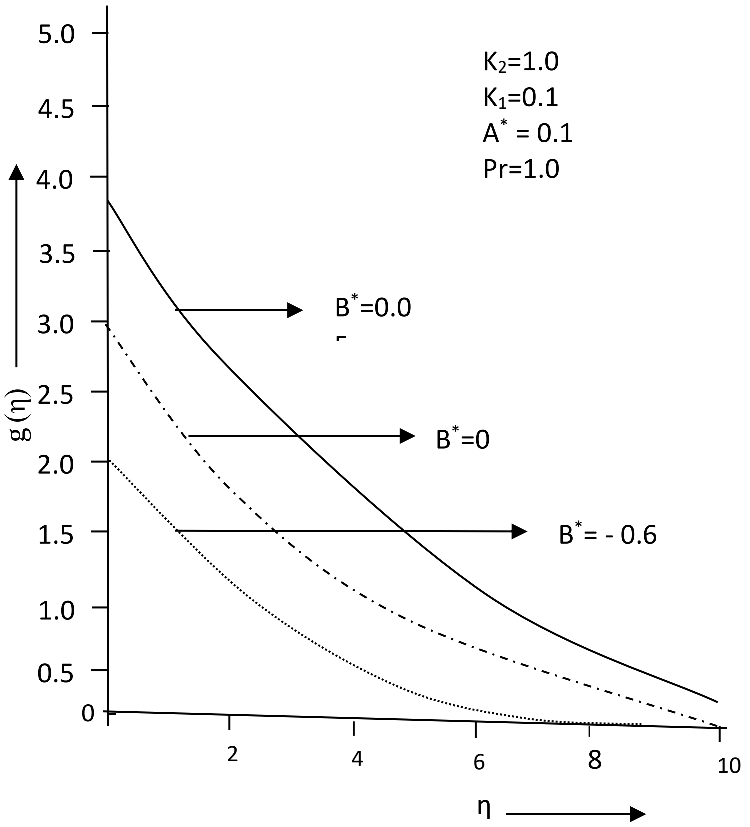

FIGURE (3b): THE GRAPH OF TEMPERATURE PROFILE g(η) Vs η FOR DIFFERENT VALUES OF TEMPERATURE DEPENDENT HEAT SOURCE/SINK PARAMETER B^* = 0.05, 0, -0.6, k_2=1.0, k_1 = 0.1, A^* = 0.1, Pr=1.0 IN PHF CASE

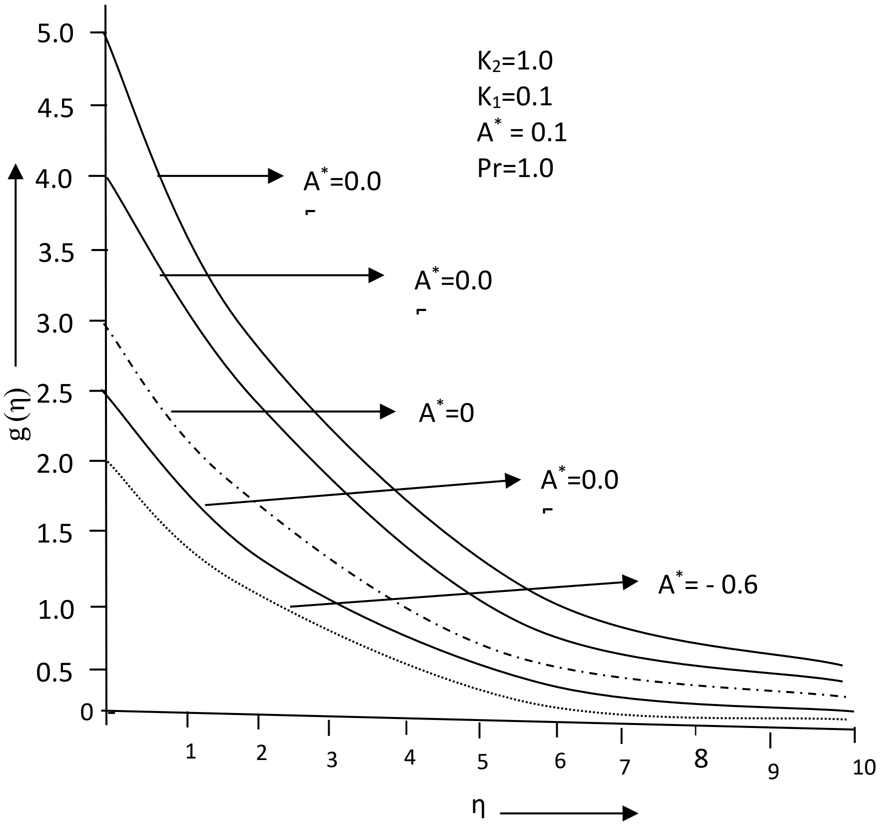

FIGURE (3c): THE GRAPH OF G(η) Vs η FOR DIFFERENT VALUES OF SPACE DEPENDENT HEAT SOURCE/SINK PARAMETER A^* = 1, 3, 0, -0.3, -0.6 WITH k_1=0.1, Pr=1.0, k_2 =1.0 IN PHF CASE

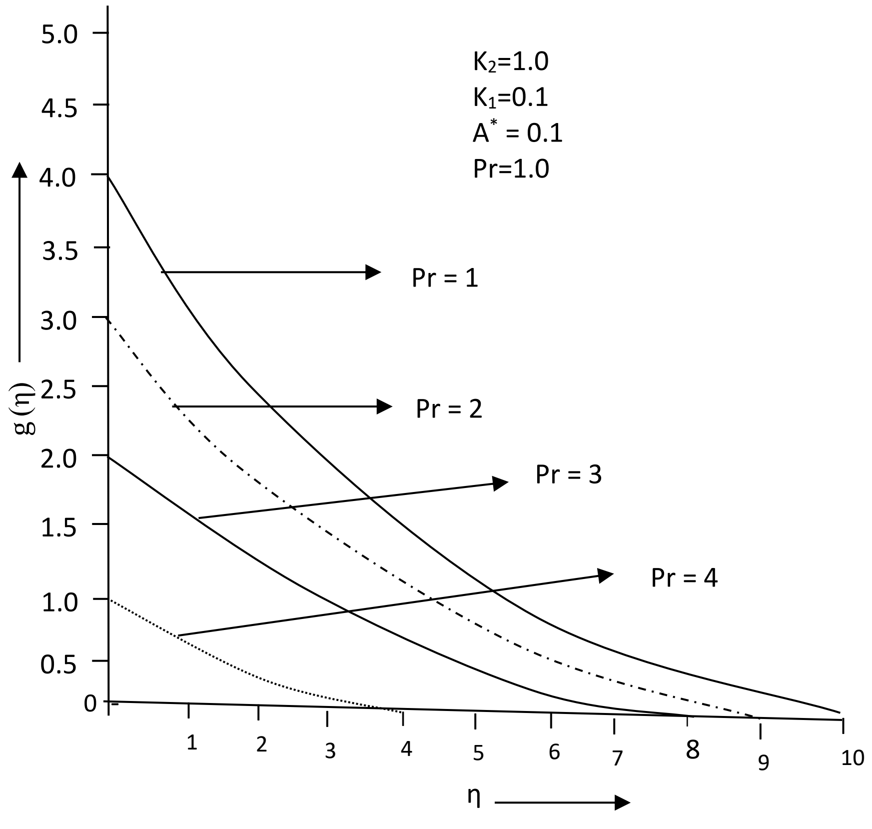

FIGURE (3d): THE GRAPH OF g(η) Vs η FOR DIFFERENT VALUES OF PRANDTLE NUMBER PARAMETER Pr = 1,2,3,4 WITH k_2 =1.0 A^* = B^*= 0.1 AND k_1=0.1 IN PHF CASE

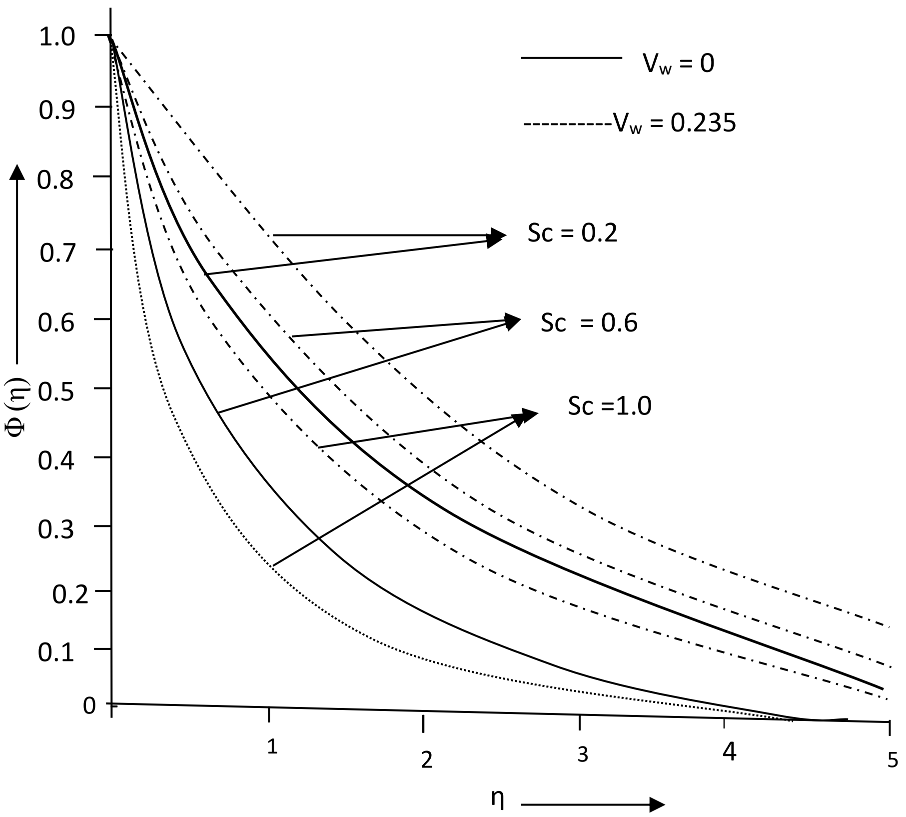

FIGURE (4a): THE GRAPH OF THE DIMENSIONLESS CONCENTRATION PROFILES Φ (η) Vs η FOR DIFFERENT VALUES OF Sc WITH k_1=0.1 , k_2 =1.0 β=0 IN PST CASE

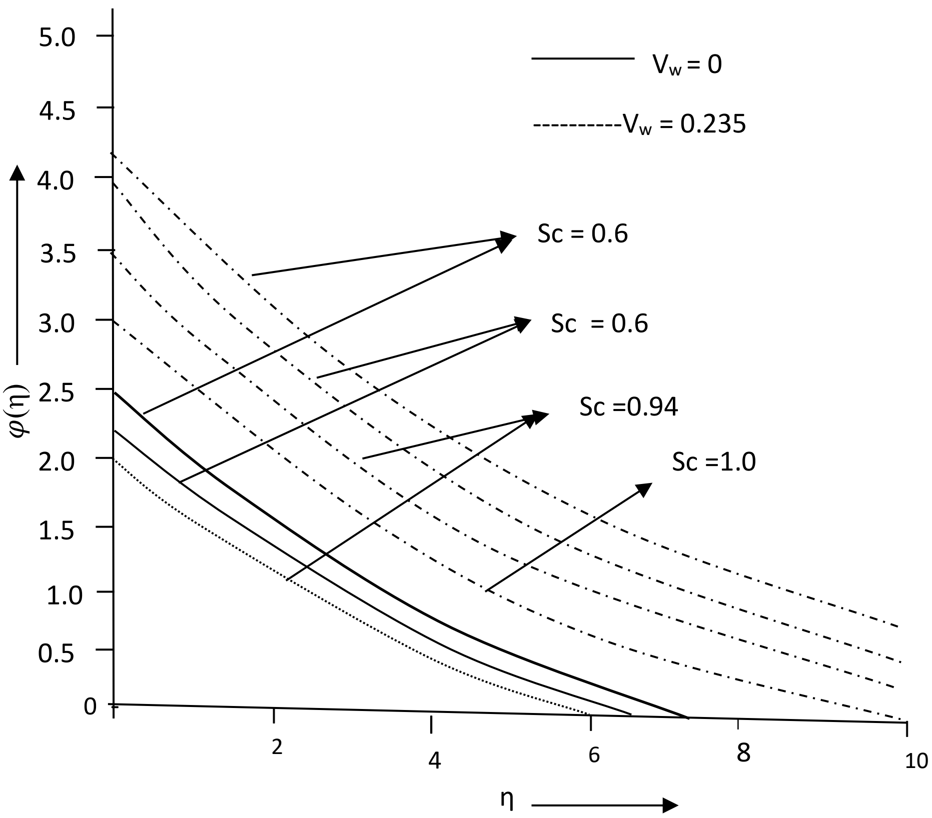

FIGURE (4b): THE GRAPH OF DIMENSIONLESS CONCENTRATION PROFILES Φ (η) Vs η FOR VARIOUS VALUES OF Sc WITH k_1=0.1 , k_2 =1.0 AND PRESCRIBED WALL MASS FLUX FOR DESTRUCTIVE FIRST ORDER reactions η = 1 IN PHF CASE

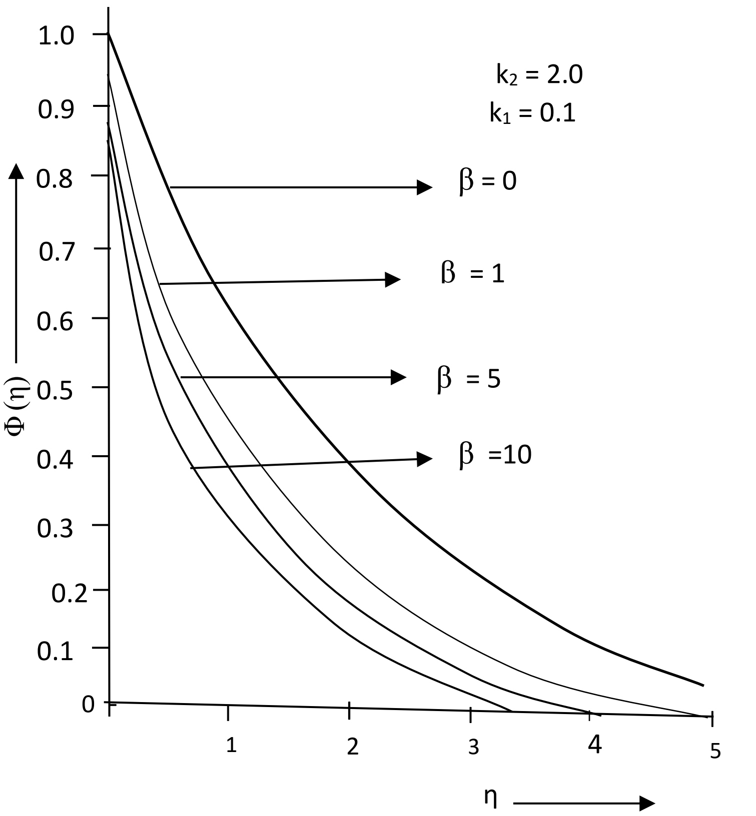

FIGURE (4c): THE GRAPH OF THE DIMENSIONLESS CONCENTRATION PROFILES Φ(η) Vs η FOR DESTRUCTIVE FIRST ORDER REACTIONS (n = 1) FOR Sc = 1 WITH THE REACTION RATE β AS PARAMETER k_1=0.1 , k_2 =2.0 IN PST CASE

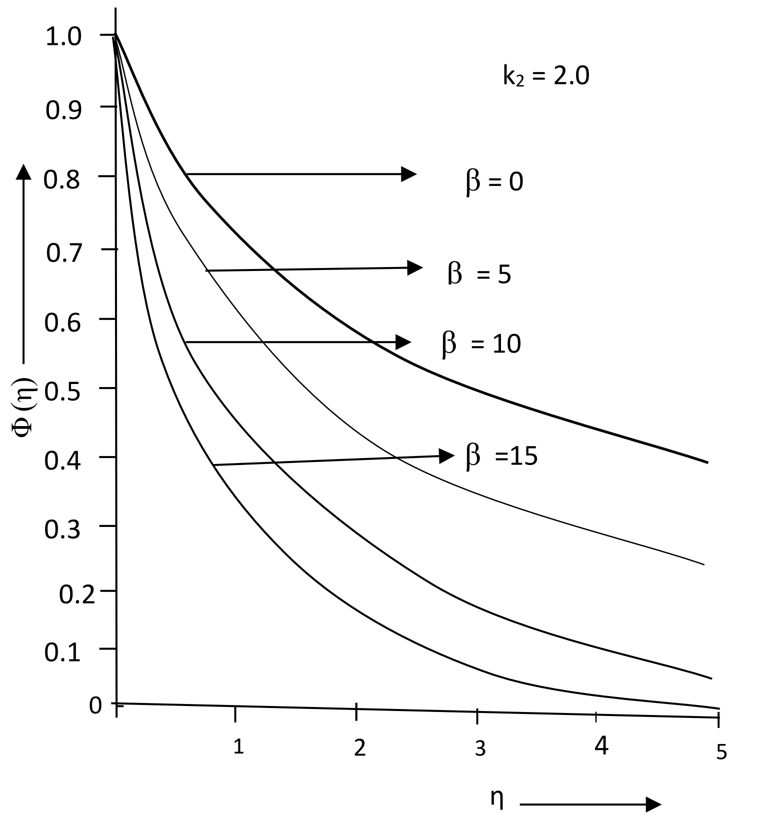

FIGURE (4d): THE GRAPH OF CONCENTRATION PROFILES Φ(η) Vs η FOR DESTRUCTIVE FIRST ORDER REACTION n =1, Sc = 0.1 WITH THE REACTION RATE β AS PARAMETER k_1=0.1.

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