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Constructability-based Multi-Objective Optimization for Reinforcing Bar Design in Rectangular Concrete Beams

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Abstract

Constructability-based optimization design of reinforcing bar (rebar) in concrete structures (RC) has been attracting attention in recent years when aiming towards industrialized sustainable construction. This paradigm enables it to more effectively link the practicality of reinforced concrete designs and their associated material usage and construction cost. The problem itself is multi-objective (MO), and the development of effective optimization algorithmic frameworks to approach its solution is essential. For this purpose, Artificial Intelligence (AI) based optimization with enhanced Meta-heuristic algorithms (MA) has demonstrated to be the key to reduce the computational demand. Particularly for beams, the deployment of Graph Neural Networks (GNN) has proven to be of the most effective AI-based optimization approaches. Nonetheless, its application for these elements has been limited, so far, to single-objective (SO) optimization and not for MO optimization, which entails further considerations to effectively reach optimal Pareto Fronts (PF) in a time-efficient manner. Additionally, the lack of constructability metrics, at this point, for rebar design in RC structures, in the literature, is still evident. Even though some efforts have been made in the last years, for some types of elements, there is still a gap when it comes to elaborate and flexible constructability models that may be used in general, for any project at hand.

This work presents the development of a novel MO optimization framework with GNN-Enhanced Metaheuristics (MA), for rebar design in multi-span beams. For this purpose, the development of a constructability score (CS) model is proposed based on rebar cuts and labor assembling complexity. The Non-Sorting Genetic Algorithm II (NSGA-II) is used for enhancement. The performance of each algorithm is analyzed and compared between Non-Enhanced, and GNN, in terms of convergence and time efficiency.

Keywords: Constructability, Multi-Objective Optimization, Graph Neural Networks, Rebar design, Reinforced Concrete Beams

1 Introduction

Optimization of rebar for RC structures has proven to be a complex problem, of such magnitude, that many researchers have referred to it to be NP-hard (Afzal M., 2020). especially for beams, columns and slabs. The use of pure meta-heuristic optimization algorithms has demonstrated, thus, to be time expensive, and without necessarily reaching acceptable optimum solutions. For RC beam elements, in comparison to RC column elements, the rebar crosses multiple design cross-section along the element's length, so that free-clash must be assured at all locations, which makes the optimization problem of higher complexity. In this regard, most of the optimization procedures developed by researchers have been focused on the section level, separately, failing in considering the parametric relationship of the various rebar layouts between such sections and between multiple structural elements. In turn, the design results usually lack the necessary and essential *constructability*, leading to non-optimized material waste and non code-compliance.(Li M. et al., 2023).

Constructability has become in recent years an essential metric for industrialized construction. It promotes the optimum use of knowledge and experience in conceptual planning and field operations, in a coordinated systematic workflow, to facilitate the construction of infrastructure designs. When considering constructability for structural design purposes, the problem is multi-objective, given the trade-off with construction costs or rebar design volumes (Lao W.-L. et al., 2023). In this context, when referring to RC structures, the development of constructability metrics, specifically for rebar designs, is of paramount importance for a more proper estimation of construction costs. Nonetheless, most of the existing research and constructability metrics, up to now, have been focused at the building level and not at the element level, ignoring the constructability of the reinforcement itself.

To solve this optimization design problem related to rebar design, meta-heuristics alone have demonstrated to be computationally expensive (Afzal M., 2020). In recent years, however, and most specifically, in the present decade, the use of Machine Learning (ML) and Deep Learning (DL) models have proven considerable efficiency to aid meta-heuristic optimization algorithms for rebar design of RC structures, both in terms of convergence and time execution. This approach, along with BIM technology, has demonstrated incredible potential with huge reductions of computational time and convergence efficiency to global optimum solutions. In (Li M. et al., 2023), for instance, Graph Neural Networks (GNN) were used to aid Exploratory Genetic Algorithms (EGA), as an automated pipeline for free-clash rebar design optimization of rebar layouts, both for structural elements and for different kinds of joints of RC buildings. It was demonstrated that under this approach, the computation time could be reduced by 75% to 90% with great convergence results. The key to this success is that, through GNN, the representation of rebar designs as graphs would enable it to consider the parametric relationship between different rebar groups in a single structural element, or among multiple ones, enhancing the efficiency of clash-free rebar design optimization.

Up to now, however, the application of AI-based optimization design has been mostly focused on single-objective (SO) optimization and not MO optimization. The applications of ANN and GNN are no exception. In fact, according to (Afshari, 2019) most of the formulations for RC beams have been SO, focused mostly on construction cost. Those few works focused on multi-objective optimization are mostly concerned with construction cost and flexural limit states, with only a couple of works focused on deflection limit states. In more recent works, a focus on constructability-based optimization has been emphasized. In (Li M. et al., 2021), for instance, a multi-objective optimization approach between material cost and labour cost was adopted. In (Lao W.-L. et al., 2023), constructability-scores of reinforcement were formulated for precast beams and columns to account for standardization in a BIM-enabled framework in a multi-objective optimization formulation with the NSGA-II-GD. Nonetheless, there seems to be an existing gap in regards to constructability-based optimization design with AI-based optimization processes. Additionally, the lack of solid formulations of constructability scores models for reinforcement designs is also evident.

This work aims at bridging the gap in the literature focused on RC beam optimization between construction cost, limit state objectives and constructability. For this purpose, a constructability score model focused entirely on the rebar layout along the whole length of beam span is proposed, by considering the number of different diameter sizes, number and distribution of the rebar itself and rebar cuts to avoid clashes. The approach to be adopted is multi-objective (constructability - construction cost / rebar volumes), to complement the current state-of-art related to AI-based optimization of rebar design in RC structures. More specifically, a GNN-Enhanced NSGA-II is proposed for single-span beams and compared to Non-Enhanced NSGA-II, in terms of time efficiency and convergence.

2 Literature review

Since the commercialization of computers, the investigation of novel optimization design methods and frameworks for engineering design started to take place. In this regard, meta-heuristics, soon, took over the conventional mathematical optimization methods given their versatility to adapt to non-linear problems of multiple variables. Perhaps the first algorithms to be used for optimization design of RC structures is the Genetic Algorithm, in the 90's. Examples of the earliest works with the GA for concrete beams are (Coello, 1997), (Koumousis & Arsenis, 2002) for beams and deep beams, considering construction costs and weight as the main objective functions.

A few years later, with the development of new meta-heuristics, hybrid meta-heuristic optimization algorithms began to be developed for the optimum design of reinforcement in RC structures, such as in (Bekdas & Nigdeli, 2012), where the Big-Bang Crunch and the Harmony Search (HS) algorithm were used for the optimum design of T-shaped beams. These advances enabled, as well, the further development of optimization processes for systems of continuous beams, as in (Govindaraj & Ramasami, 2005) where the GA was used, based on rebar templates and prototypes. In later years, BIM technology began to be used, along with these optimization processes, to work as the visualization platform for better appreciation of the optimum results and automation of detail drawings, and eventually to serve as the programming platform itself, as in (Li, M. et al., 2021).

In most of these studies, however, due to the high complexity of the problem and search space of rebar solutions, many simplifications and assumptions have been formulated for the optimization processes to have relatively acceptable efficiency in time execution (Shaqfa & Orban, 2019). Thus, most of the solutions found usually turn out to be far from the global optimum ones. In (Govindaraj & Ramasami, 2005), for instance, the authors used reinforcement tables for each beam cross-section, separately, without considering the rebar as an optimization variable, limiting the feasible search space to what the database would provide. Similar approaches have been adopted in other works, as in (Kwak & Kim, 2008), where no global optimum solutions are ever found, but only improved ones from a given initial proposed solution. The main issue lies on identifying the key features related to the relationship between the rebar designs of the different critical cross-sections along a beam span or several spans. That is where ANN/GNN and ML/DL, in general, can have the greatest advantages.

3 Constructability analysis for rebar in concrete beams

Constructability refers to the optimum use of construction knowledge and experience in the conceptual planning, engineering, procurement and field operations phases to achieve the overall project objectives. When referring to rebar design in RC structures, the number of rebars and its distribution as well as the rebar diameter sizes are the two main factors to consider. For this purpose, the specification of rebar design patterns or prototypes should be created. These patterns may vary, according to the design mechanism of each structural element. For beams, for instance, it is common to have different rebar layers in the tension zone, one on top of another. In this work, the rebar prototype is shown in Figure 1. For such rebar design prototype, the following enlisted distribution constraints are considered.

1. A maximum of three layers of rebars in tension is considered for each design cross-section
2. Each layer can be only composed of two rebar diameter sizes
3. Any rebar on the upper layers has to be placed right above the bottom layer

A minimum of two bars in tension and two in compression should always be provided, for confining reinforcement.

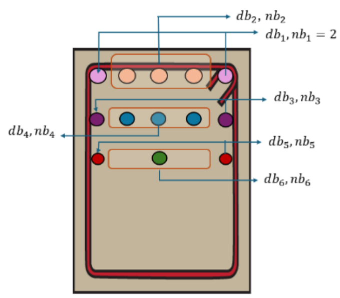

Figure 1: Rebar design prototype for a beam cross-section.

However, specifically for beams, additional factors, in relation to other structural elements, should be also considered. For instance, the number of cuts to perform would be of great influence on the constructability of the element. In general, cuts are usually sought to minimize, to reduce waste. Additionally, free-clash constraints must be satisfied at all costs, both at the element level as well as when considering multi-span beams with joints between them.

3.1 Constructability Score model

In summary, for beams, three main variables should be considered when measuring the constructability of rebar designs: uniformity of number of rebars and their distribution *UNB*, uniformity of rebar diameter sizes *UND* and number of rebar cuts *NC*, as expressed next in (1):

$$CS = UNB + UND + NC \tag{1}$$

The variable *UNB* is the uniformity of number of rebars distributed over the main three design cross-sections (left, middle and right). This variable depends on the number of rebar layers *nlay* at each cross-section *i*, as following. The maximum number of layers at each cross-section is considered as three [$1 \leq nlay_i \leq 3$]:

$$UNB_i = \begin{cases} \frac{1}{(nlay_i)^{W_{UNB}}} \sum_{j=1}^{nlay_i - 1} UNBL_{i,j}\,, & nlay > 1 \\ 1, & nlay = 1 \end{cases} \tag{2}$$

In the previous equation, the variable $UNBL_j$ refers to the uniformity of number of rebars $nb_{j,i}$ between each layer j of the cross-section i, with respect to the bottom layer $nlay_{1,i}$, as follows:

$$UNBL_{i,j} = \begin{cases} \frac{nb_{i,j+1}}{nb_{i,1}}, & 2 \leq nb_{i,j} \leq nb_{i,1} - 1 \\ 1, & otherwise \end{cases} \tag{3}$$

The weight factor $W_{\{UNB\}}$ penalizes the number of layers, so that the UNB score may decrease as the number of layers increase. Thus, it is recommended for this weight factor to be $[1 \leq W_{\{UNB\}} \leq 2]$. In general, for each cross-section $[0 < UNB_i \leq 1]$.
The variable UND refers to the uniformity of rebar diameter sizes of each cross-section, and is defined as follows:

$$UND_i = \frac{1}{ND_{i,j}^{W_{\{ND\}}}} \tag{4}$$

where $ND_{i,j}$ is the number of different rebar diameter sizes $[1 \leq ND_{i,j} \leq 3]$. Because the ND score decreases as the number of rebar diameter sizes increase, it is recommended that the corresponding weight factor W_{ND} is between $[0.1 \leq W_{ND} \leq 1]$.

Finally, the variable UC refers to the uniformity in which rebar cuts at each cross-section take place. It is defined as following:

$$UC = \begin{cases} \frac{1}{nlay_i^{W_{NC}}} \sum_{j=1}^{nlay_i} \sqrt{\frac{nb_{i,j} - nbc_{i,j}}{nb_{i,j}}}, \ 1 \leq nb_{i,j} \leq nb_{i,1} - 1 \\ 1, nlaycut_i = 0 \end{cases} \tag{5}$$

As it could be observed, the corresponding weight factor of this last variable W_{NC} penalizes the number of layers in which cuts are going to take place $nlaycut_i$, instead of the number of cuts per layer or per section. The reason is that usually, the complexity of executing cuts on rebars would depend more likely on the number of layers that require cuts, rather than the number of cuts themselves. This weight factor is recommended to have values between $[0.1 \leq W_{NC} \leq 1]$.

4 Optimization framework

4.1 Design constraints

According to the ACI 318-19 (ACI318, 2019) beams of special moment frames should comply with the following design constraints, both in regard to dimensions and reinforcement quantity:

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1. Maximum and minimum rebar cross-section area:

$$A_{s,min} \geq \begin{cases} (\frac{3\sqrt{f_c'}}{f_y} b_w d \\ \frac{200}{f_y} b_w \end{cases} \tag{6}$$

$$A_{s,max} \leq 0.0025bh \tag{7}$$

2. Minimum rebar separation

For parallel nonprestressed reinforcement in a horizontal layer, clear spacing shall comply with:

$$sep_{min,x} = max \begin{cases} 25mm \\ d_b \\ \frac{4}{3} d_{agg} \end{cases} \tag{8}$$

For parallel reinforcement placed in two or more horizontal layers, reinforcement in the upper layers shall be placed directly above reinforcement in the bottom layer with a clear spacing between layers of at least $sep_{min,y} \geq 25mm$.

3. Side rebars must be provided, in case the height dimension is greater than $900mm$. For such case, they should be distributed uniformly for a distance $h/2$ from the tension face. Spacing of this skin reinforcement should not be in accordance to (9) where $f_s = 2/3 f_y$.

$$sep_{min,y} = max \begin{cases} 15\left(\frac{40000}{f_s}\right) - 2.5C_c \\ 12\left(\frac{40000}{f_s}\right) \\ 250mm \end{cases} \tag{9}$$

4.2 AI-based optimization design process

The relative new tendency of AI-based optimization design in engineering considers that the training of the ML/DL model is done before executing any optimization process. In order to take the most advantage of the AI model, the engineering process itself must be decomposed in two or more stages, so that the AI model may be used, in as many as possible ones, for leverage.

The whole optimization design process is summarized as shown below in Figure 2, both for the Non-AI-Enhanced optimization algorithm with pure MAs (left panel), and for the AI-based optimization design process with MAs (right panel). In general, the process for both paradigms consists of a three-stage hybrid optimization process with the PSO and GA. In total, twelve main optimization variables take place. Such variables are the rebar diameter sizes and number of rebars for the left and middle section of the beam, over their respective three layers in tension. After rebar cut design takes place, additional analogous optimization variables are considered for the right cross-section. In this case, the DL/ML model is used in two sub-stages of the general optimization design process, namely in (1) design of reinforcement in left and middle cross-sections and (2) design of reinforcement in right cross-section, after cut design.

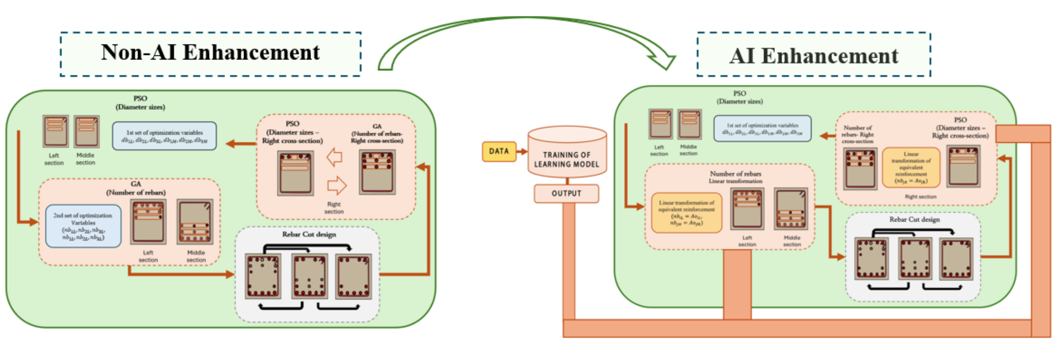

Figure 2: Summarized optimization design process of rebar in rectangular concrete single-span beams.

4.3 Constructability-based MO optimization

When formulating a constructability-based MO optimization design of structures, the constructability is sought to be maximized, while minimizing at the same time, the construction costs or material usage, as shown next in Figure 3. When the problem is stated in terms of construction cost, it can be expressed mathematically as in (10), where x is the decision vector is composed by the $n = 12$ variables just mentioned previously (namely the discrete variables corresponding the rebar diameters for the left and middle cross-section). S is the feasible objective space with $m = 2$ objective functions, namely the total construction cost C_s of reinforcement $f_1(x)$ and the constructability CFA $f_2(x)$, under the $i = 1, \dots, q = 5$ constraints just mentioned (max and min rebar separation, structural resistance efficiency and max/min rebar cross-section area).

$$\begin{gathered} \min\,[F(x) = (f_1(x), 1 - f_2(x)] \\ S.T.: x = [x_1, x_2, \dots, x_n] \in S \\ Under: g_i(x) \leq y_i, i = 1,2, \dots, q \end{gathered} \tag{10}$$

For this work, a hybrid three-staged MO optimization algorithm with the NSGA-II was formulated, as shown next in Figure 3.

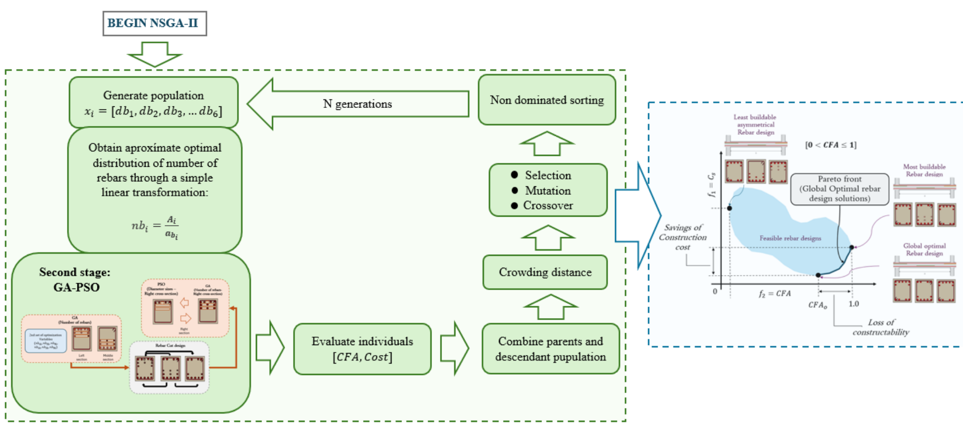

Figure 3: Flow diagram depicting the formulated non-enhanced three-staged hybrid NSGA-II for MO optimization design of rebar in concrete beams.

4.4 Generation of training data

Differing from what happens in AI-Enhanced SO optimization, where only one optimum solution is sought for convergence, in AI-Enhanced MO optimization it has been suggested in the literature to work with Multi-Levels of Optimum Targets (M-LOT). That is, to generate training data sets through non-enhanced MO Optimization, with pure meta-heuristics, for instance, and then, extract multiple optimum solutions from the PFs to generate multiple output entries for the training data. For this specific problem, this formulation of AI training is depicted next in Figure 4, where, for instance, three-LOT are considered. The input data, on the other hand, consists of six entries, namely: cross-section dimensions b, h concrete's compressive strength f'_c, span's length L and pure bending load actions M_{uL}, M_{uM}, M_{uR}.

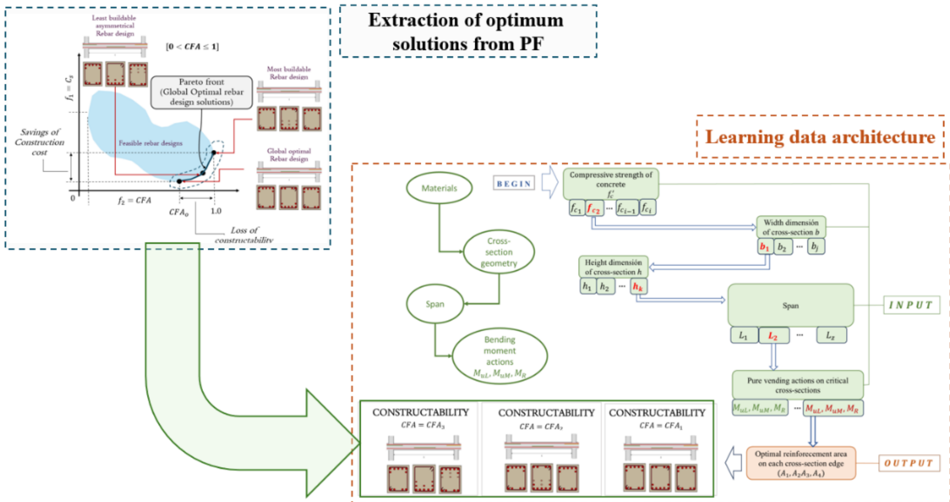

Figure 4: Generation of training data sets for AI-based optimization of rebar in beams, through MLOT.

4.5 AI-Enhanced optimization design framework

Based on the previous formulations, the Non-enhanced MO optimization framework from Figure 3 would be transformed into the one shown next in Figure 5 under an AI-based paradigm. Here, given that M-LOT are considered for the training of the ML/DL model, multiple models are created for each constructability level of rebar design. Therefore, in this MO AI-based optimization process, when generating an individual, a random selection process takes place to choose a constructability level of rebar design, to predict the required amount of rebar cross-sectional area for each cross-section. This process gives each generation more diversity of individuals, and therefore, better approximations to the optimum PF. The default paper size is US letter, but A4 or letter sizes are also acceptable.

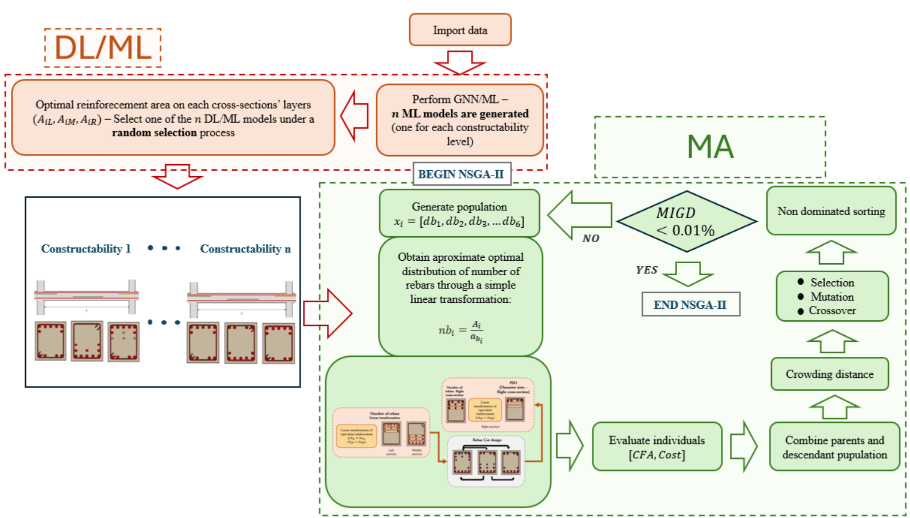

Figure 5: Proposed AI-Enhanced constructability-based MO optimization framework for the design of rebar in concrete beams through MLOT.

4.6 Graph Neural Networks representation for rebar design in beams

The versatility of GNN have made them widely popular in applied AI for multiple disciplines, given that many phenomena can be represented as graphs. Particularly, for rebar design in concrete beams, the rebar prototyping of each cross-section is represented by a node, and the relationship between each cross-section design as edges, as shown next in Figure 6. In previous studies, related to application of GNN for this problem of rebar design in beams, usually three critical cross-sections are considered for each span, as it was also the case in this work. Each node consists of six features. Features related to b, h, L, f_c' are considered to be constant for each node in the graph representation, where the other features vary for each node, namely the critical design bending moments at left, middle and right span cross-sections $M_{u-left}, M_{u-mid}, M_{u-right}$.

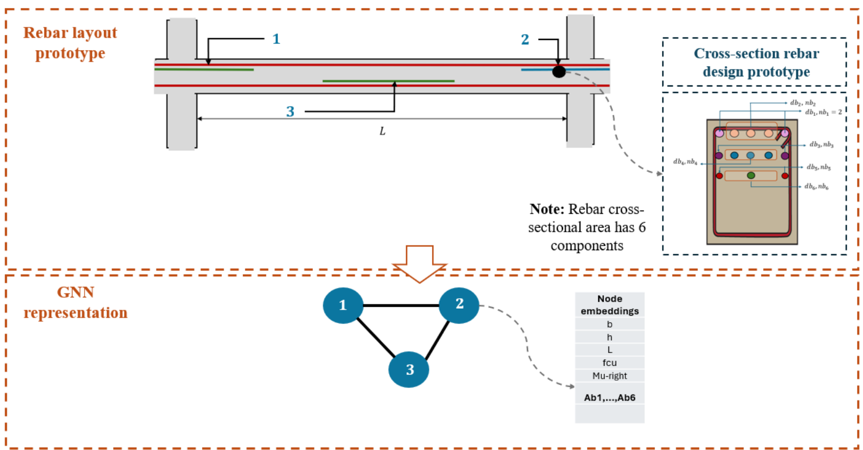

Figure 6: Architecture of the GNN for rebar representation in single-span concrete beams.

5 Results

5.1 Showcase of the framework: convergence and time efficiency.

To showcase the proposed optimization formulation, the following beam models of Figure 7 and Figure 8 are considered:

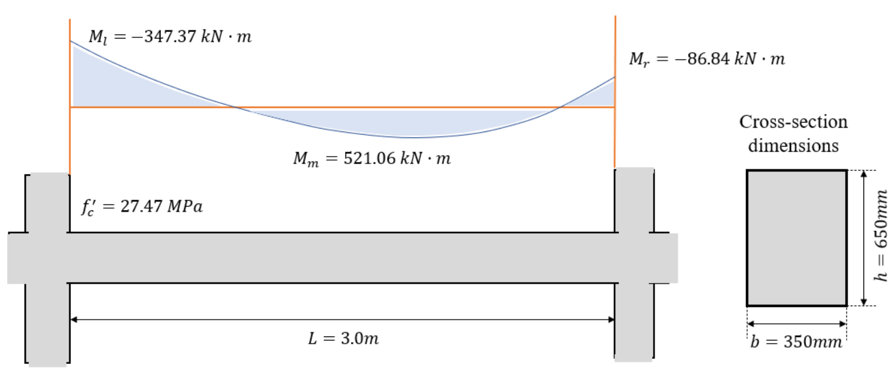

Figure 7: Experimental beam model 1 for showcase of the proposed optimization design framework.

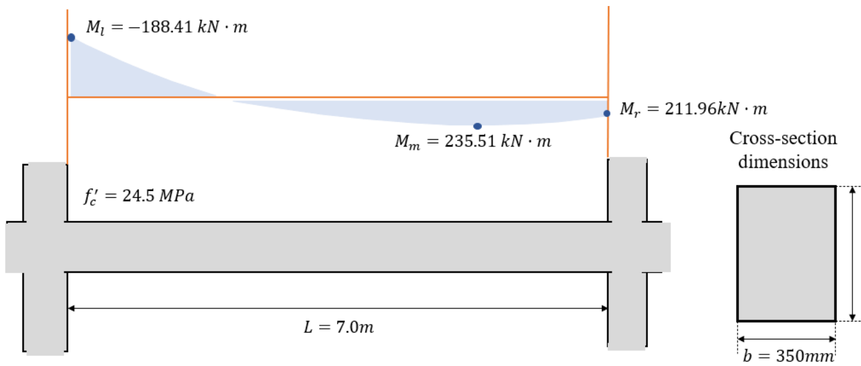

Figure 8: Experimental beam model 2 for showcase of the proposed optimization design framework.

The final PF with Non-ML Enhanced NSGA-II, for each beam model, are shown below:

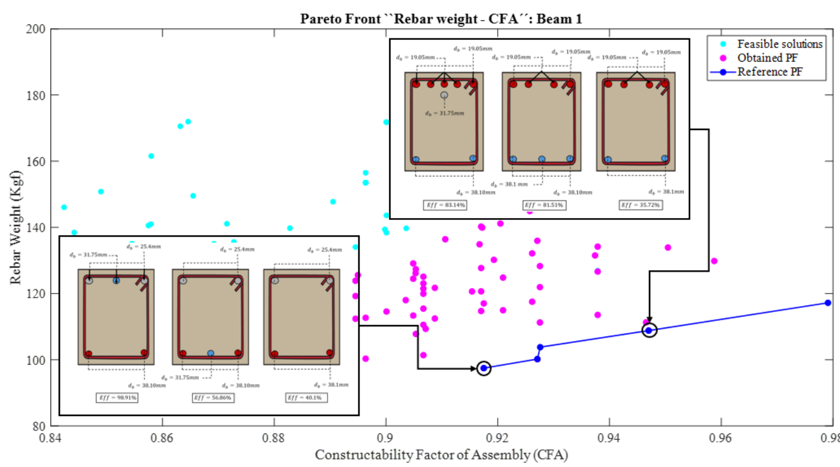

Figure 9: Final PF with Non-enhanced NSGA-II, for beam model 1.

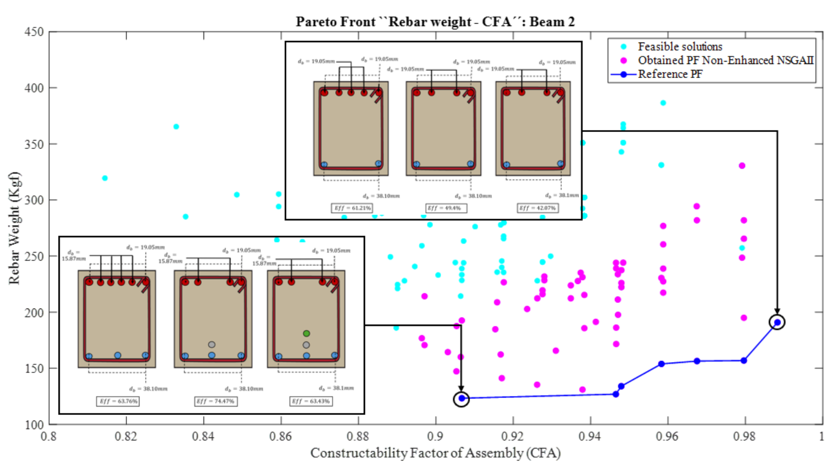

Figure 10: Final PF with Non-enhanced NSGA-II, for beam model 2.

The respective evolution of convergence of each Non-enhanced NSGA-II, for each beam model, are shown next, in Figure 11:

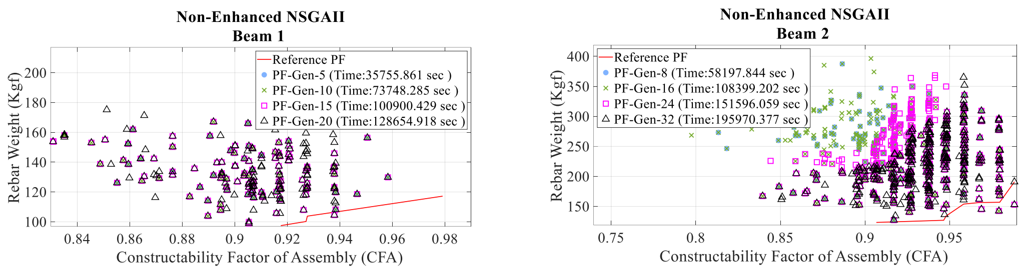

Figure 11: Evolution of convergence of Non-Enhanced NSGA-II. Left panel, for beam model 1. Right panel, for beam model 2.

5.2 Performance of optimization framework.

To assess and compare the performance of the framework with the ML enhancement (GNN-MA), a dataset with a total of 4000 samples was generated. A relation 7/3 was established for training/testing. The Accuracy and MSE are shown as follows in Figure 12.

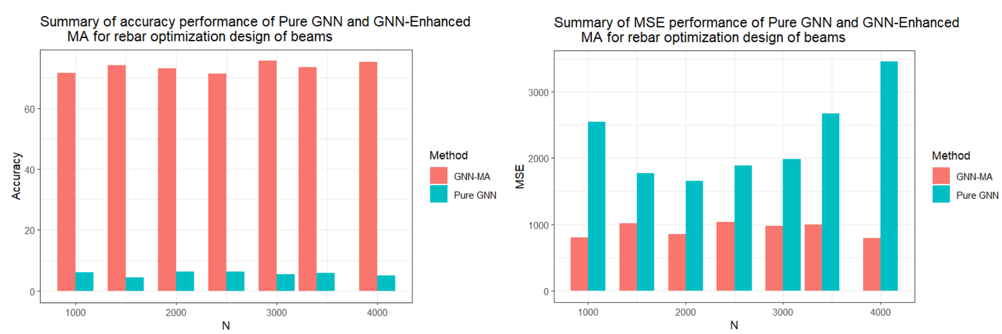

Figure 12: Comparison of Accuracy and MSE between GNN-MA and pure GNN, for different sizes of datasets.

A comparison of convergence efficiency between Non-Enhanced NSGA-II and GNN-Enhanced NSGA-II is shown below, in terms of Mean Inverted Generational Distance (MIGD) and time complexity.

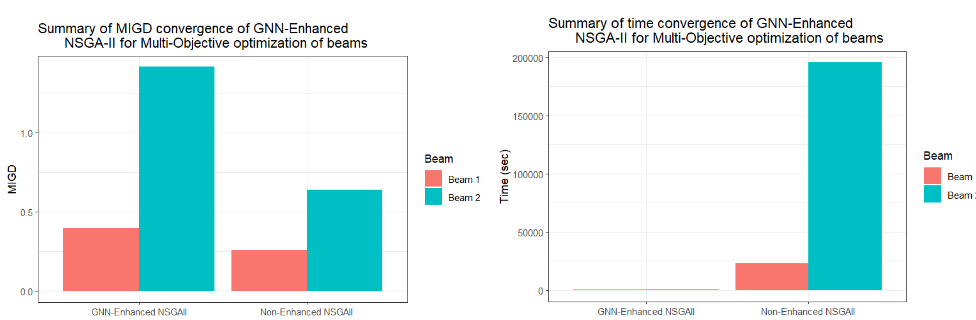

Figure 13: Experimental beam model 2 for showcase of the proposed optimization design framework.

The evolution of the PFs in time, for each beam model is shown below in Figure 14 and Figure 15, respectively, for different MLOT.

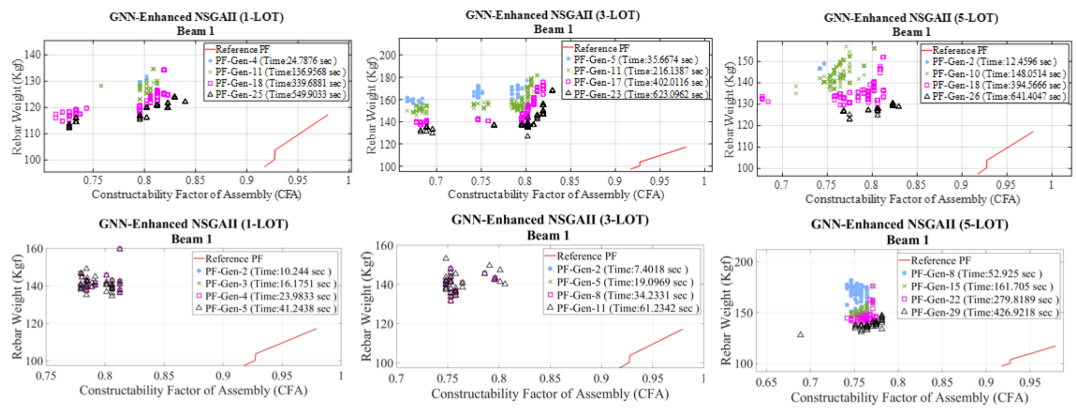

Figure 14: Experimental beam model 2 for showcase of the proposed optimization design framework.

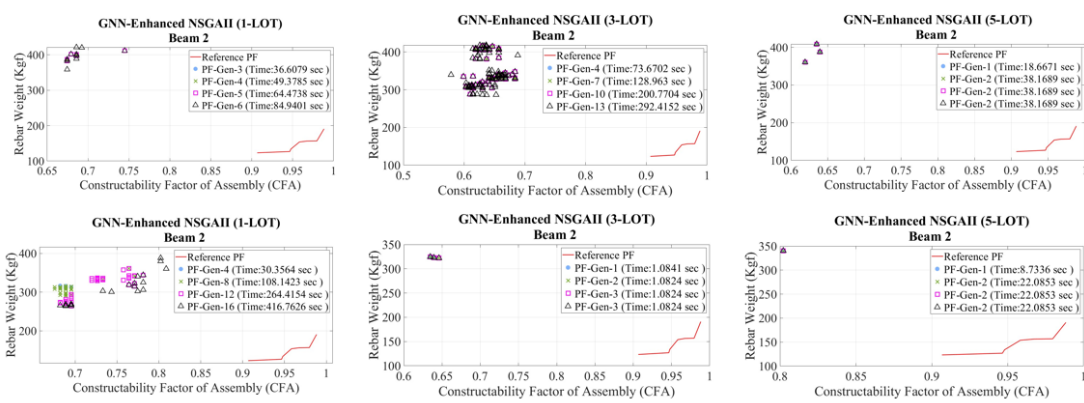

Figure 15: Experimental beam model 2 for showcase of the proposed optimization design framework.

6 Discussion

The Algorithmic framework for SOO of rebar design in beams turned out to be effective. Increases of accuracy levels of as much as approx. 91%, in average, and reductions of MSE of approx. 77%, were demonstated, in comparison to pure GNN, as observed in Figure 12.

However, for MOO, the heavy compuational demand for the generation of efficient databases of optimum reinforcement in beams, through MLOT, makes it challenging for the GNN-NSGAII to generate close approximations to global optimum PFs. This issue can be observed in Figure 14 and Figure 15, corresponding to the convergence in time through different MLOT, where the obtained optimum PF are clearly distant from the global optimum ones.

In this sense, to improve the results, the following could be done: (1) generate a larger database of at least 10,000 samples, (2) increase the Number of population and generations for the Non-enhanced NSGA-II to generate data samples, using high performance computers, (3) develop more efficient surrogates for MOO, through different DL techniques, embedded simultaneously into the NSGA-II or any other evolutionary MOO algorithm. For instance, Graph Attention Networks (GAT) could be used for classification of feasible or non-feasible solutions during population generation. The use of generative AI models, such as Generative Adversarial Networks (GAN) or diffusion models could also be an option to generate more data during the optimization process, when the data size generated is limited.

7 Conclusions

A novel ML-based MO optimization algorithmic framework for reinforcement bar design of rectangular concrete beams, based on constructability, was proposed in this work. In short, the proposed MOO design framework turned out to be effective, under the philosophy of digital fabrication and construction. The advantages obtained in terms of time execution with the proposed surrogate were major, in terms of computational demand, with savings of approximately 99% of the time needed for a Non-ML Enhanced MO optimization algorithm. However, in terms of convergence the proposed surrogate assisted evolutionary algorithm demonstrated to be limited, requiring expensive optimization simulations for the generation of data to effectively train ML/DL models.

In relation to the proposed Constructability Score (CS) model for rebar designs in rectangular single-span concrete beams, savings of material usage of approximately 15.25% and 34% were observed for beam model 1 and 2, respectively, with reductions of constructability of only 10% and 8%, respectively. In general, the proposed model was able to represent well the complexity and ease of construction of rebar designs in rectangular beams, as they become more complex.

8 Acknowledgements

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