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Benchmark: DC-to-DC Switched-Mode Power Converters (Buck Converters, Boost Converters, and Buck-Boost Converters)

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Abstract

Power electronics represent a large and important class of hybrid systems, as modern digital computers and many other systems rely on their correct operation. In this benchmark description, we model three DC-to-DC switched-mode power converters as hybrid automata with continuous dynamics specified by linear ordinary differential equations. A DC-to-DC converter transforms a DC source voltage from one voltage level to another utilizing switches toggled at some (typically kilohertz) frequency with some duty cycle. The state of this switch gives rise to the locations of the hybrid automaton, and the continuous variables are currents and voltages. The main contributions of this benchmark description include: (a) unified modeling of three types of converters as a hybrid automaton with two locations and differing continuous dynamics, and (b) a basic benchmark generator that allows for simulation of these converters in Simulink/Stateflow and reachability analysis in SpaceEx. Future challenges for these benchmark classes include closed-loop control, where the speeds of plant and controller dynamics differ by orders of magnitude.
Category: academic **Difficulty:** medium (open-loop); challenge (closed-loop)

1 Context and Origins

DC-to-DC power converters are frequently implemented using switches (power transistors) for efficiency reasons, and switched-mode power supplies are commonly used in digital computers and many safety-critical systems, from cars and airplanes to medical devices and industrial control systems. A DC-to-DC power converter transforms an input (source) voltage level to an output (load) voltage level. These systems are naturally hybrid due to their switching, and have been studied using hybrid systems tools, such as in [2–6, 9, 10].

We present academic versions of these benchmarks using idealized models, but they represent an important class of case studies for industrial usage as power electronics are increasingly controlled using software. For example, the reported root cause of the 2014 recall of $700,000$ model year $2010 - 2014$ Toyota Priuses that could stall during driving is an interaction between a boost converter and its software controller (emphasis and footnote added) [7]:

> “Inside the inverter assembly is an Intelligent Power Module (IPM) which contains a control board equipped with transistors known as Insulated-Gate Bipolar Transistors

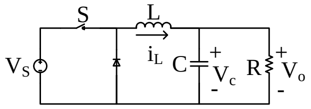

Figure 1: Buck converter circuit—a DC input V_s is decreased to a lower DC output V_o.

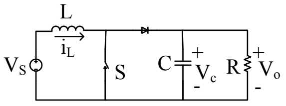

Figure 2: Boost converter circuit—a DC input V_s is increased to a higher DC output V_o.

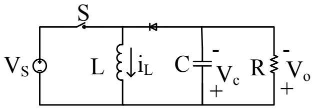

Figure 3: Buck-boost converter circuit—a DC input V_s is increased or decreased to a higher or lower DC output V_o.

> (IGBTs). *Due to certain characteristics of the software used to control the boost converter in the IPM, higher thermal stress could occur in specific IGBT's used for the operation of the boost converter*, which is required during high-load driving such as accelerating during highway driving. If this occurs, the IGBT could deform and eventually result in damage to the IGBT(s), illuminating various warning lights on the instrument panel. In most cases, the vehicle will enter a fail-safe mode, resulting in reduced motive power in which the vehicle can still be driven for certain distances. In limited instances, the motor/generator ECU could reset, causing the hybrid system[1] to shut down and resulting in the vehicle stopping while being driven, increasing the risk of a crash."

We briefly review switched-system models of buck, boost, and buck-boost converters, but the interested reader is referred to in-depth derivations of these models and non-idealized versions in power electronics textbooks such as [1,8]. See Figures 1, 2, and 3 for circuit diagrams of the buck, boost, and buck-boost converters, respectively. A buck converter takes an input voltage of say 5V and "bucks" or drops the voltage to some lower DC voltage, say 2.5V. Conversely, a boost converter takes an input voltage of say 2.5V and "boosts" or raises the voltage to some higher DC voltage, say 5V. Based on the duty cycle of the transistor's switching, a buck-boost converter can either increase or decrease a (potentially varying) source voltage, although this necessitates closed-loop control that we do not address here, but discuss in future challenges.

2 Brief Description

These converters may each be described by a hybrid automaton with two modes, where each mode corresponds to the state of the switch S (open/closed state of S in each of Figures 1, 2, and 3). Mode switches from open to closed occur every $D \cdot T$ seconds and from closed to open every $(1-D)T$ time, where D is the duty cycle and $T = \frac{1}{f}$ is the switching period corresponding to a switching frequency f. The converters' specifications are to transform a source voltage V_s to a desired reference voltage V_{ref} so that $|V_{ref} - V_o|$ is zero, where $V_{ref} < V_s$ for the buck converter, $V_{ref} > V_s$ for the boost converter, and either $V_{ref} < V_s$ or $V_{ref} > V_s$ for the buck-booster converter.

Open-Loop Converters: In open-loop control (i.e., without comparing V_o to V_{ref}), mode switches occur as just described for a *constant* duty cycle D, picked at design time based on considerations such as expected load impedance, etc. In contrast, closed-loop control is typically achieved with pulse width modulation (PWM) and the duty cycle D varies as a function of V_o.

The primary reachability safety properties to check are:

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[1] Referring to the hybrid motor/engine drive.

Component / Parameter Name	Symbol	Range
Input Voltage	V_s	$[11.95, 12.05]$ V
Desired Output Voltage	V_{ref}	5 V
Actual Output Voltage	V_o	5 V $\pm \epsilon$
Load Resistance	R	$[0.95, 1.05]\Omega$
Capacitor	C	$[23.75, 26.25]$ uF
Inductor	L	$[47.5, 52.5]$ uH
Switching Period	T	$[24.5, 25.5]$ us
Switch-open duty cycle	$1 - D$	0.4
Switch-closed duty cycle	D	0.6

Table 1: Example buck-converter parameter values and variations (see Figure 4).

Converter	Switch S State	A_m	B_m	Duty Cycle D
Buck	open	$\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\frac{V_{ref}}{V_s}$
	closed	$\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$	
Boost	open	$\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$	$\frac{V_{ref}-V_s}{V_{ref}}$
	closed	$\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$	
Buck-Boost	open	$\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\frac{V_{ref}}{V_{ref}+V_s}$
	closed	$\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$	

Table 2: Dynamics of the three converters.

1. start-up regulation: from the initial condition $V_o = V_C = 0$V and $i_L = 0$A, after some time T_o, the output voltage is near the desired output voltage ($|V_{ref} - V_o| \leq \epsilon$ for some small $\epsilon > 0$) (see Figure 4 where the initial set of states were $i_L \in [0, 0.1]$A and $V_C \in [0, 0.1]$V),
2. steady-state regulation: once in steady-state (i.e., from the initial condition $V_o = V_{ref}$ and corresponding i_L), the output voltage remains near the desired output voltage ($|V_{ref} - V_o| \leq \epsilon$ for some small $\epsilon > 0$).

3 Key Observations

Several features of these power converter systems can serve to evaluate reachability algorithms and tools, such as:

1. Switching Frequency: In open-loop, timing determines when switches occur, so this benchmark serves to evaluate how tools handle time-triggered transitions.

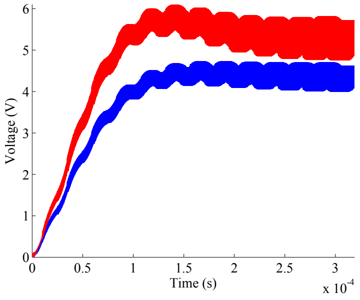

Figure 4: Start-up verification with SpaceEx's LGG algorithm of open-loop buck converter output voltage using parameters from Table 1. The red and blue sets overapproximate different parameter extrema [4].

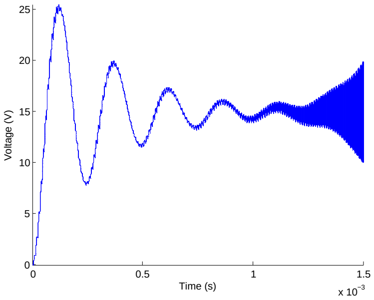

Figure 5: Start-up verification with SpaceEx's STC algorithm of open-loop buck-boost converter where $V_s = 20$V is decreased to $V_o = 15$V. This illustrates challenges from eigenvalues with non-negative real part.

2. Instantaneous or Urgent Transitions: As the duty cycle D approaches either 0 or 1, the time spent in either the closed or open states also goes to zero, which may make reachability computations challenging. See Figure 6 for clarification, in these limits, the model does not simply become a linear system, although this is an artifact of the modeling due to the invariants.
3. Non-negative eigenvalues: The buck converter is easy to analyze as in all modes, all its eigenvalues have strictly negative real part (see Figure 4). However, the boost and buck-boost converters have eigenvalues with non-negative real part (particularly one zero), which may make reachability analysis challenging, for instance, as illustrated by the blow-up upon stabilizing near the desired output voltage of 15V in Figure 5.
4. Closed-loop challenges: In closed-loop, the duty cycle D is selected as a function of the output voltage V_o. For typical closed-loop controllers (see, e.g., the closed-loop controllers in [3]), the controllers may be modeled as a linear system ($\dot{x} = Ax$), but dynamics of the controller are typically much faster than those of the plant (the circuit). Reachability analysis that relies on a uniform time step may struggle with either being too coarse (time advancement in too large of steps) or too slow (time advancement sufficiently small, but reachability analysis takes too long) [3].

4 Outlook

In future work, we intend to further investigate closed-loop control to address issues that arise in closed-loop linear controllers described in more depth in [3]. The basic issue is that the controller dynamics are significantly faster—an order of magnitude or more—than the plant dynamics, which leads to issues in determining appropriate parameters for the reachability analysis. The analysis is either too coarse and meaningless due to overapproximation errors with a large sampling time, or takes too long due to too fine a choice of sampling time. Additionally, non-idealities such as parasitics, temperature variations, switch delays, discontinuous conduction mode, non-ideal diodes, etc., can be integrated into these models to make them more realistic and of potential use in industrial scenarios. Adding such non-idealities may alleviate some issues (e.g., parasitics may remove the zero eigenvalue in the buck-boost and boost converters), but will also introduce new challenges like nonlinear dynamics and additional continuous variables.

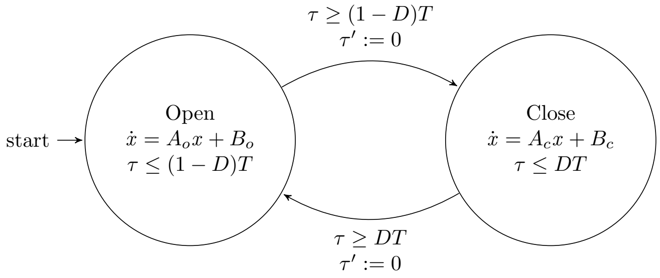

Figure 6: Hybrid automaton model of the three converters (see the A_m matrices, B_m vectors, and duty cycles D in Table 2).

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A Appendix

Each of the buck, boost, and buck-boost converters have two real-valued state variables modeling the inductor current i_L and the capacitor voltage V_C, depicted for the respective converters in Figures 1, 2, and 3. While the circuit topologies are different, each are characterized using these state variables, written in vector form as: $x = [i_L; V_C]$. Standard realizations of these converters utilize PWM to switch between the connected and disconnected modes, so an additional real-valued state variable τ models real-time ($\dot{\tau} = 1$). The dynamics of the continuous variables in each mode $m \in \{\mathit{Open}, \mathit{Close}\}$—in short, o and c—are specified as linear (affine) differential equations: $\dot{x} = A_m x + B_m u$, where $u = V_s$. Figure 6 shows a hybrid automaton model for the three converters. The A_m matrices consist of $L > 0$, $R > 0$, $C > 0$ real-valued constants, respectively representing inductance (in Henries), resistance (in Ohms), and capacitance (in Farads). See Table 2 for the A_m matrices and B_m vectors that specify the dynamics for the three converters. Simulink/Stateflow models, hybrid automata models with particular parameter choices (suitable for SpaceEx), and a basic benchmark generator written in Matlab with output to SpaceEx supporting parameter variations of the electrical components are available in the included files, and are posted online at `http://cps-vo.org/group/ARCH/` and `http://www.taylortjohnson.com/research/nguyen2014arch.zip`.